



**NiPS** Laboratory  
Noise in Physical Systems



# Physics of Information

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Igor Neri - July 17, 2018

NiPS Summer School 2018

Energy aware transprecision computing



# Limits to computation

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- Minimum size of computing device
- Maximum computational speed of a self-contained system
- Information storage in a finite volume
- Energy consumption limit to:
  - computation
  - memory preservation

Minimum size of computing device

# Transistor count

Processor	Transistor count	Date of introduction	Designer	Process	Area
TMS 1000	8,000	1974 <sup>[3]</sup>	Texas Instruments	8,000 nm	11 mm <sup>2</sup>
Intel 4004	2,300	1971	Intel	10,000 nm	12 mm <sup>2</sup>
Intel 8008	3,500	1972	Intel	10,000 nm	14 mm <sup>2</sup>
MOS Technology 6502	3,510 <sup>[4]</sup>	1975	MOS Technology	8,000 nm	21 mm <sup>2</sup>
Motorola 6800	4,100	1974	Motorola	6,000 nm	16 mm <sup>2</sup>
Intel 8080	4,500	1974	Intel	6,000 nm	20 mm <sup>2</sup>
RCA 1802	5,000	1974	RCA	5,000 nm	27 mm <sup>2</sup>
Intel 8085	6,500	1976	Intel	3,000 nm	20 mm <sup>2</sup>

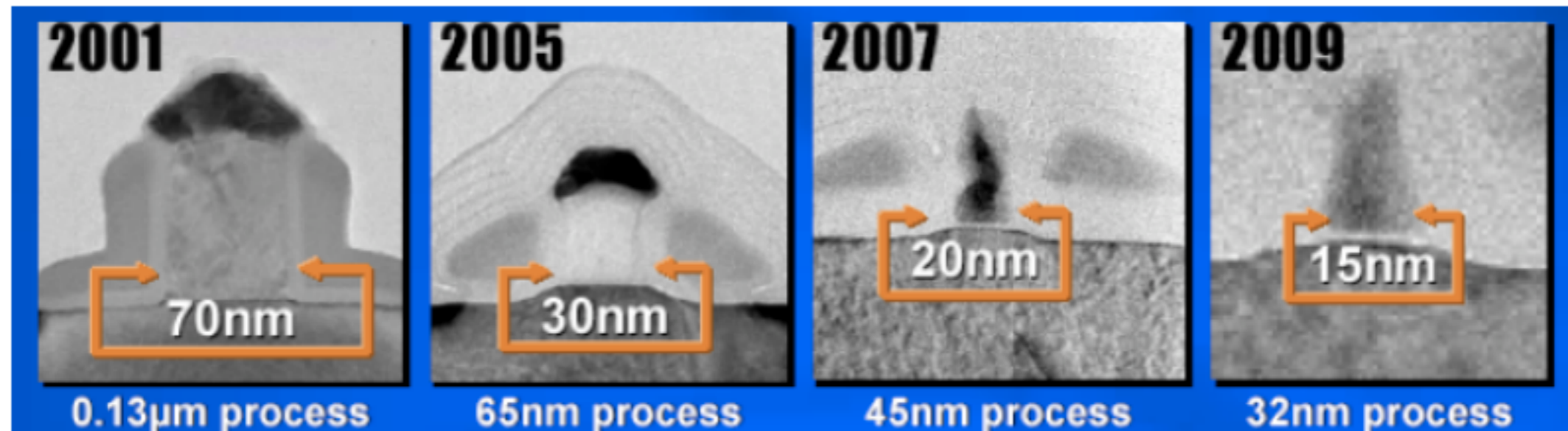
...

61-core Xeon Phi	5,000,000,000 <sup>[38]</sup>	2012	Intel	22 nm	720 mm <sup>2</sup>
Xbox One main SoC	5,000,000,000	2013	Microsoft/AMD	28 nm	363 mm <sup>2</sup>
18-core Xeon Haswell-E5	5,560,000,000 <sup>[39]</sup>	2014	Intel	22 nm	661 mm <sup>2</sup>
IBM z14	6,100,000,000	2017	IBM	14 nm	696 mm <sup>2</sup>
Xbox One X (Project Scorpio) main SoC	7,000,000,000 <sup>[40]</sup>	2017	Microsoft/AMD	16 nm	360 mm <sup>2</sup> <sup>[40]</sup>
IBM z13 Storage Controller	7,100,000,000	2015	IBM	22 nm	678 mm <sup>2</sup>
22-core Xeon Broadwell-E5	7,200,000,000 <sup>[41]</sup>	2016	Intel	14 nm	456 mm <sup>2</sup>
POWER9	8,000,000,000	2017	IBM	14 nm	695 mm <sup>2</sup>
72-core Xeon Phi	8,000,000,000	2016	Intel	14 nm	683 mm <sup>2</sup>
IBM z14 Storage Controller	9,700,000,000	2017	IBM	14 nm	696 mm <sup>2</sup>
32-core SPARC M7	10,000,000,000 <sup>[42]</sup>	2015	Oracle	20 nm	
Centriq 2400	18,000,000,000 <sup>[43]</sup>	2017	Qualcomm	10 nm	398 mm <sup>2</sup>
32-core AMD Epyc	19,200,000,000	2017	AMD	14 nm	768 mm <sup>2</sup> (4 x 192 mm <sup>2</sup> )



# Transistor size

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“Alcune riflessioni sulla legge di Moore”, Roberto Saracco, Future Center, TILAB

In terms of size [of transistors] you can see that we're **approaching the size of atoms** which is a fundamental barrier, but it'll be two or three generations before we get that far - but that's as far out as we've ever been able to see. We have another 10 to 20 years before we reach a fundamental limit. By then they'll be able to make bigger chips and have transistor budgets in the billions. - G. Moore

Energy limits speed of computation

# Energy limits speed of computation

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- What limits the laws of physics place on the speed of computation?



# Energy limits speed of computation

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Heisenberg uncertainty principle

$$\Delta E \Delta t \geq \hbar/2$$

**wrong interpretation:** it takes time  $\Delta t$  to measure energy to an accuracy  $\Delta E$

**right interpretation:** a quantum state with spread in energy  $\Delta E$  takes time at least

$$\Delta t = \pi \hbar / 2 \Delta E$$

to evolve to an orthogonal (and hence distinguishable) state

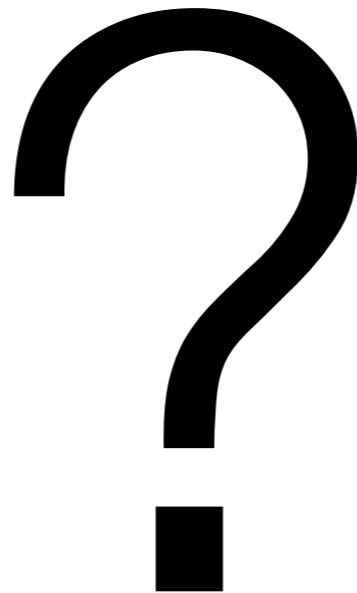
# Maximum computational speed of a self-contained system

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- **Bremermann's Limit** is the maximum computational speed of a self-contained system in the material universe. It is derived from Einstein's mass-energy equivalency and the Heisenberg uncertainty principle, and is  $c^2/h \approx 1.36 \times 10^{50}$  bits per second per kilogram
- The **Margolus–Levitin theorem** gives a fundamental limit on quantum computation (strictly speaking on all forms on computation). The processing rate cannot be higher than  $6 \times 10^{33}$  operations per second per joule of energy

# Maximum computational speed of a laptop

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# Maximum computational speed of a laptop

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- If the mass is **m** then  **$E = mc^2$**
- $m = 1 \text{ Kg}$ ,  $E = 1 (3 \cdot 10^8)^2 = \text{approx} = 10^{17} \text{ J}$
- $\Delta t = \text{approx} = 10^{-34} / 10^{17} = 10^{-51} \text{ s}$

# Comparison with existing computers

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- Conventional laptops operate much more slowly than the ultimate laptop
- Two reasons for this inefficiency:
  - most of the energy is locked up in the mass of the particles of which the computer is constructed
  - a conventional computer employs many degrees of freedom for registering a single bit

Memory space limits

# Memory space limits

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- The amount of information that a physical system can store and process is related to the number of distinct physical states accessible to the system
- A collection of **M** two-state systems has  **$2^M$**  accessible states and can register **M** bits of information
- A system with **N** accessible states can register  **$\log_2 N$**  bits of information

# Memory space limits

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- The number of accessible state, **W**, of a physical system is related to its thermodynamic entropy by the formula:  
 **$S = k_B \log W$**
- The amount of information that can be registered by a physical system is  **$I = S(E)/k_B \log 2$**
- **S(E)** is the thermodynamic entropy of a system with expectation value for the energy **E**



# Information storage in a finite volume

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- The **Bekenstein bound** limits the amount of information that can be contained within a given finite region of space which has a finite amount of energy:

$$S \leq \frac{2\pi kRE}{\hbar c}$$

$$I \leq \frac{2\pi cRm}{\hbar \ln 2} \approx 2.577 \times 10^{43} mR$$

# Information storage in a finite volume

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- Human brain

$$I \leq \frac{2\pi c R m}{\hbar \ln 2} \approx 2.577 \times 10^{43} m R$$

- mass  $m=1.5$  kg
- volume of  $1260 \text{ cm}^3$
- approximating volume to a sphere  $R = 6.7$  cm
- $I = 2.6 \times 10^{42}$  bits
- $O = 2^I$  states of the human brain must be less than  $\approx 10^{7.8 \times 10^{41}}$

# Comparison with existing computers

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- The amount of information that can be stored by the ultimate laptop  $\approx 10^{31}$  bits
- Conventional laptops can store  $\approx 10^{12}$  bits
- This is because conventional laptops use many degrees of freedom to store a bit where the ultimate laptop uses just one
- There are considerable advantages to using many degrees of freedom to store information, stability and controllability being perhaps the most important

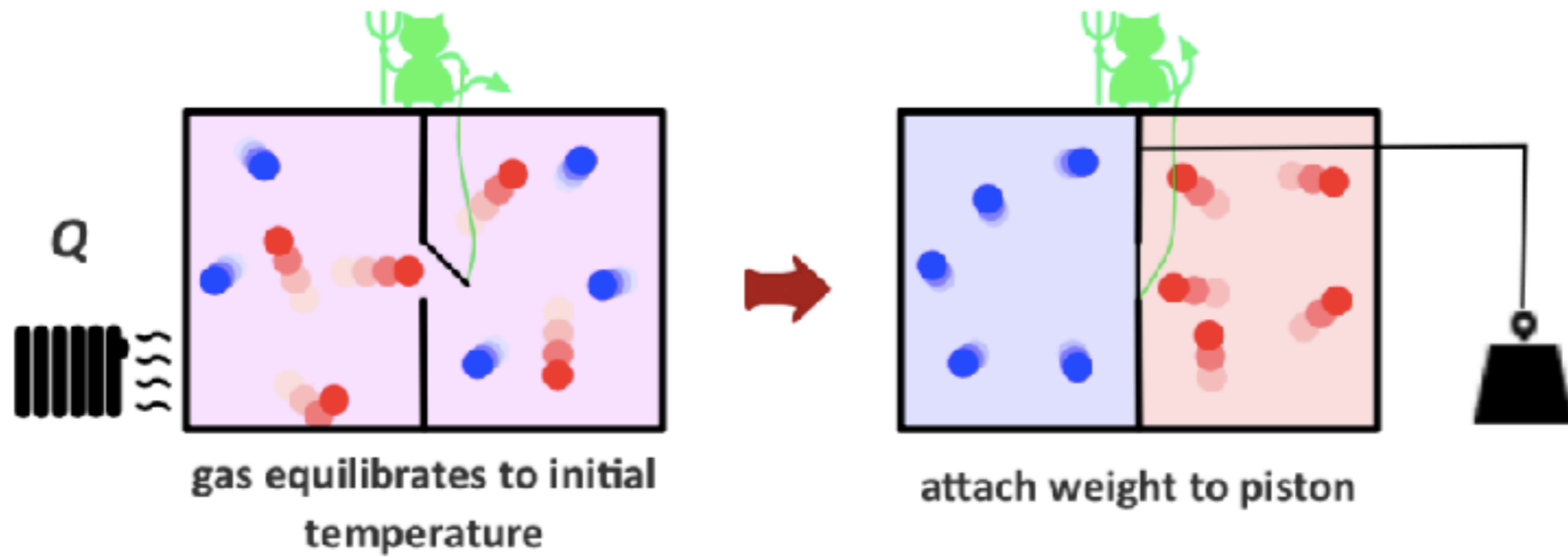
Minimum energy consumption for computation



*Information  
is  
physical*

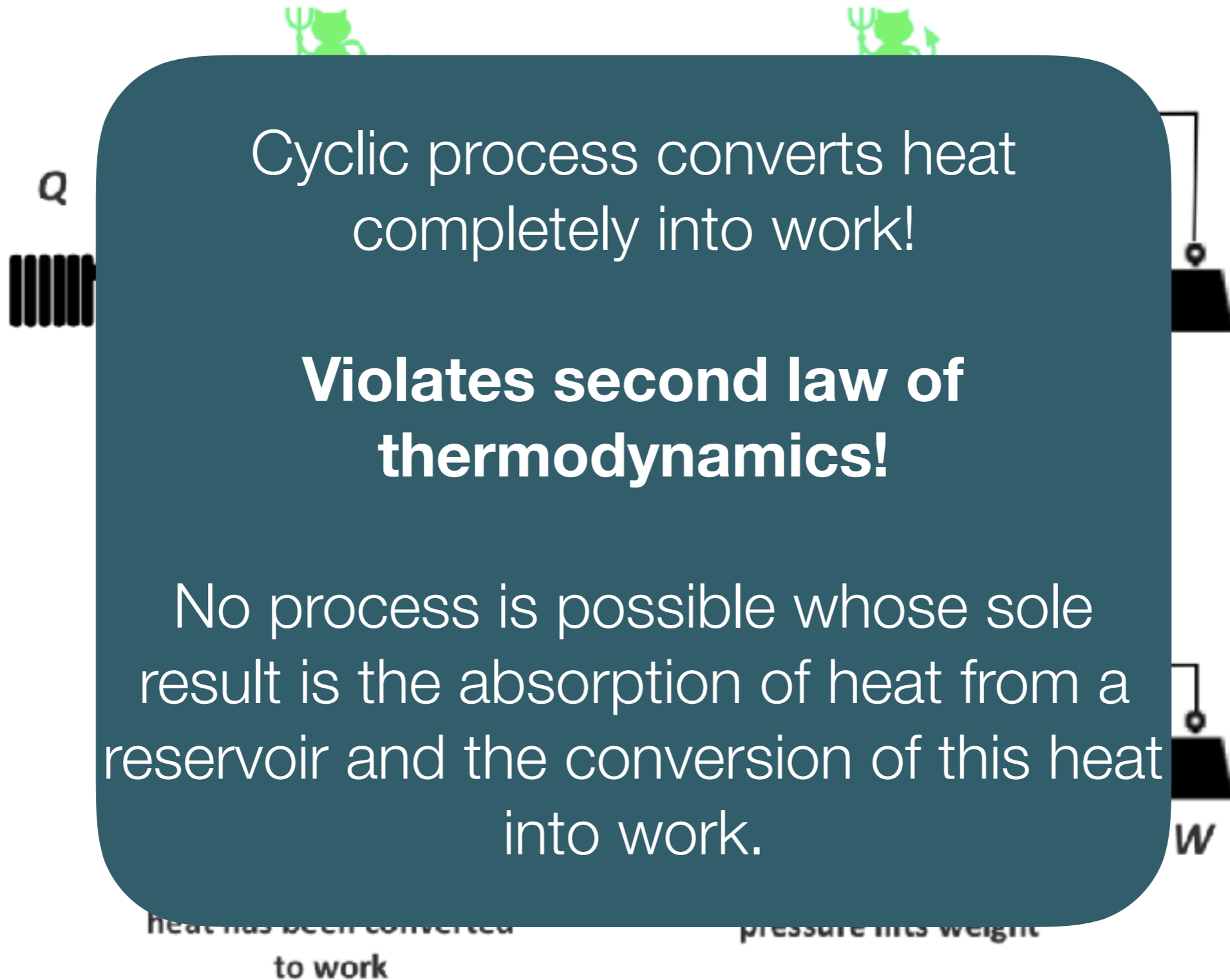
# Maxwell's demon

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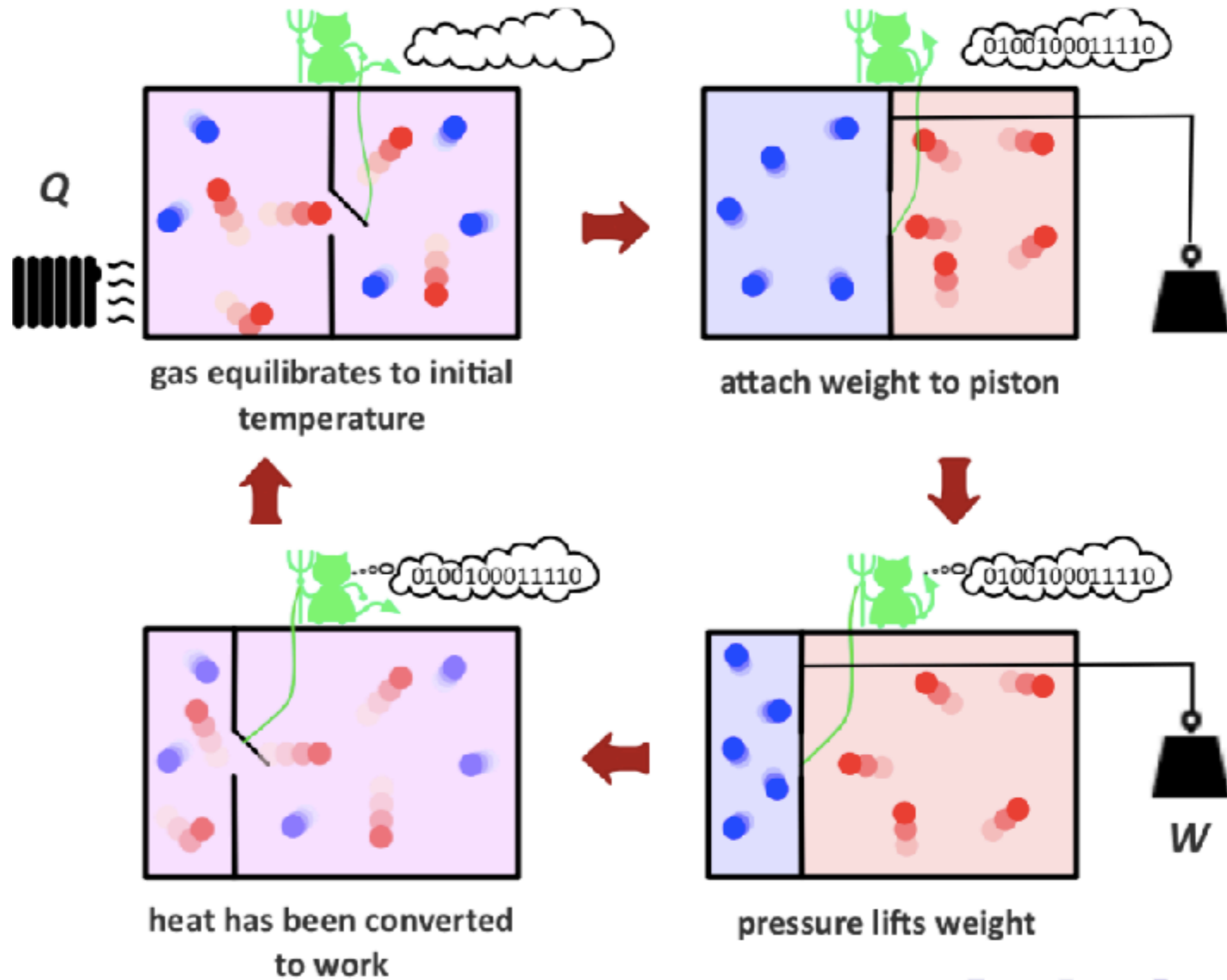


# Maxwell's demon

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# Maxwell's demon





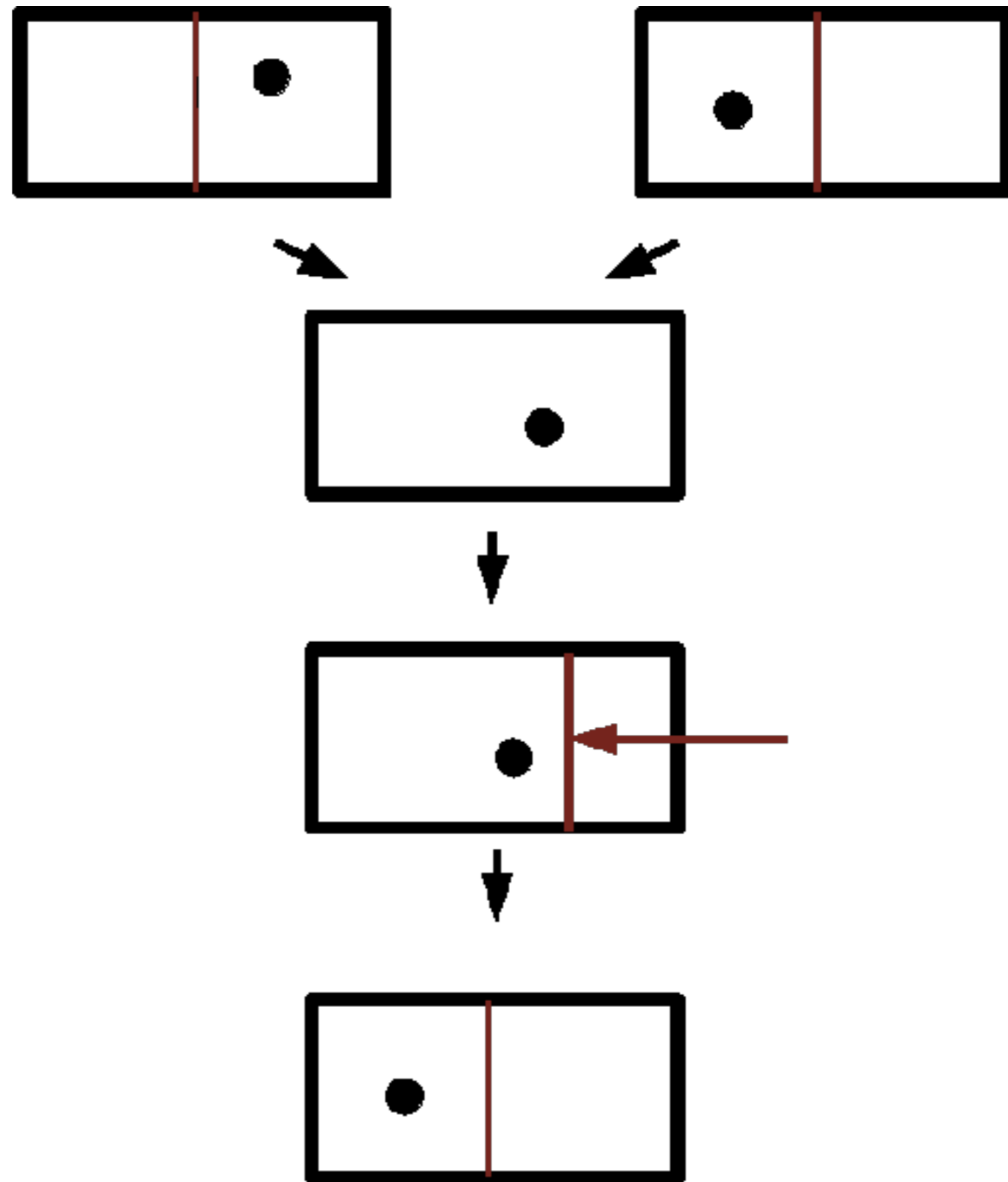


*Information  
is  
physical*

What happens when computation is logically irreversible?

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- Minimum amount of energy required greater than zero
- Let assume the operation of bit reset
- # of initial states: 2
- # of final states: 1



# Landauer principle

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- Initial condition: two possible states  
 $S = k_B \log W$

$$Q \leq T \Delta S$$

- Final condition: one possible state

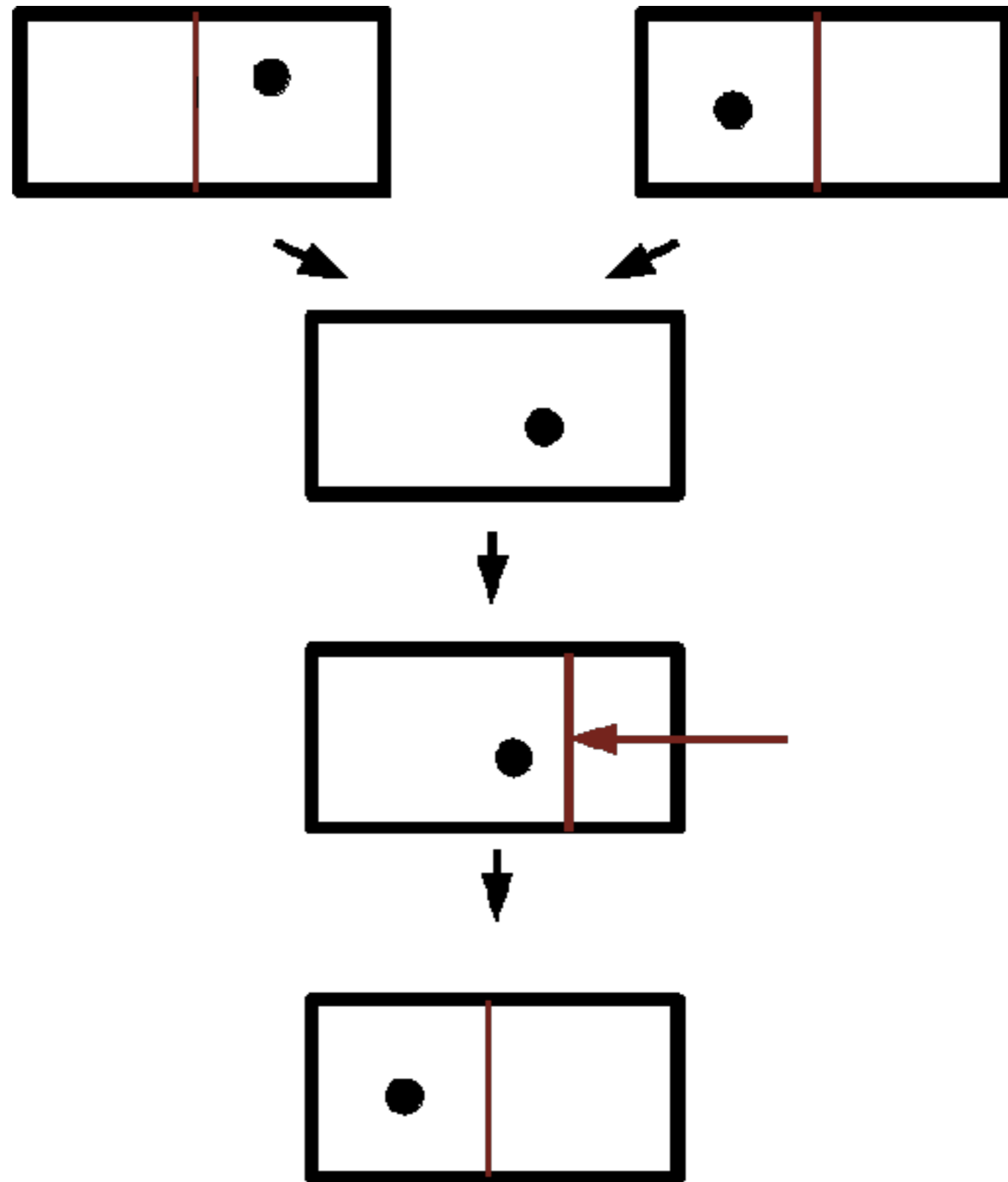
$$S_i = k_B \log 2$$

$$S_f = k_B \log 1$$

$$\Delta S = S_f - S_i = -k_B \log 2$$

- Heat produced

$$Q \leq T \Delta S = -k_B T \log 2$$



# Landauer principle at room temperature

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$$Q \leq T\Delta S = -k_B T \log 2 \sim 10^{-21} J$$

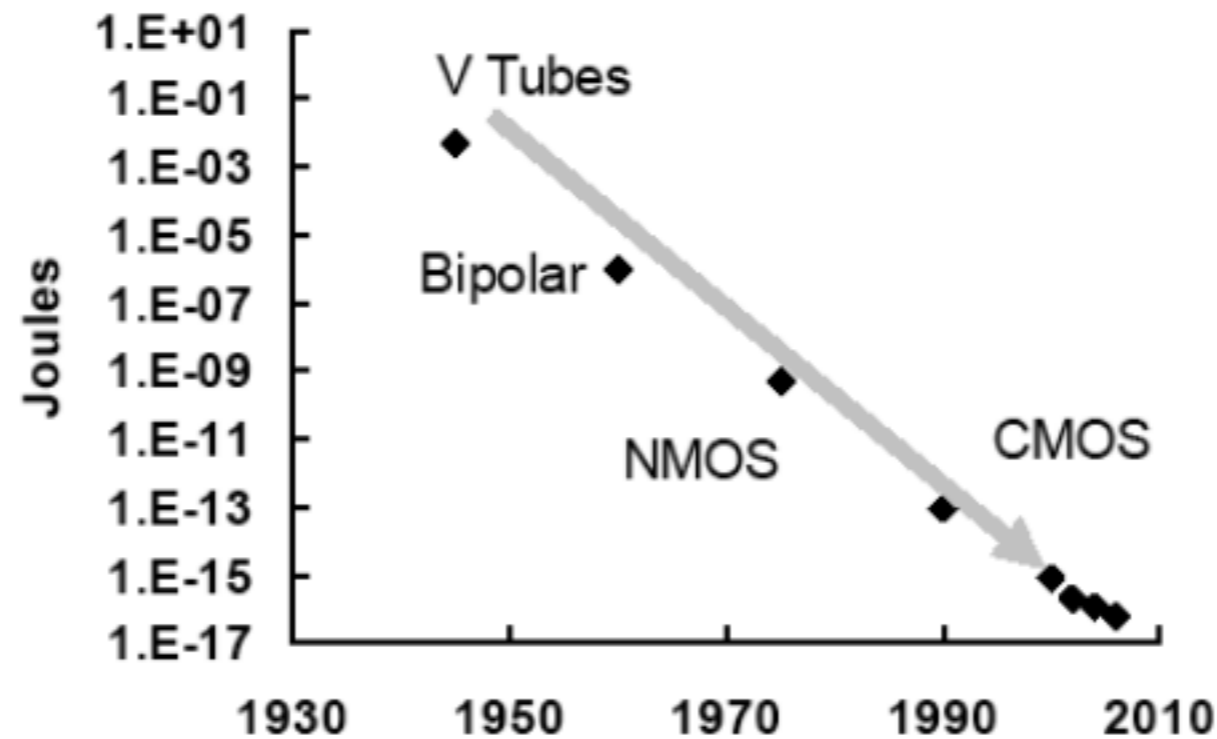
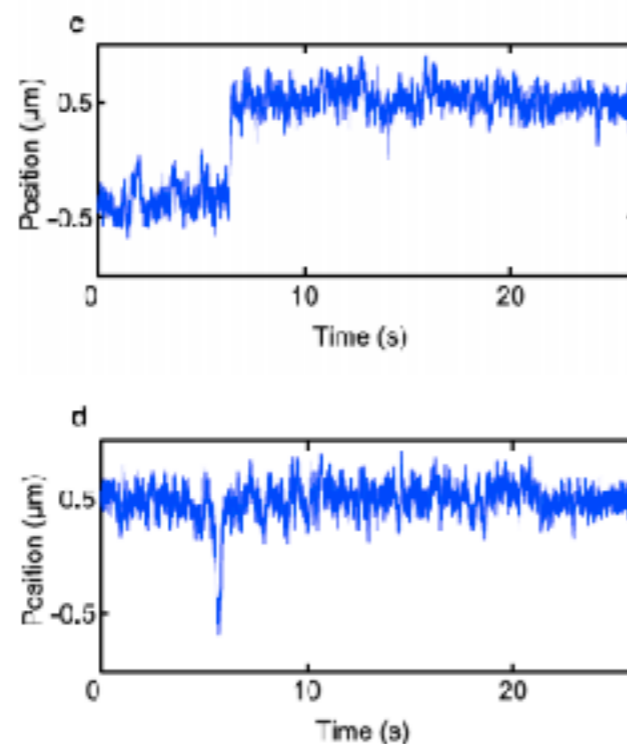
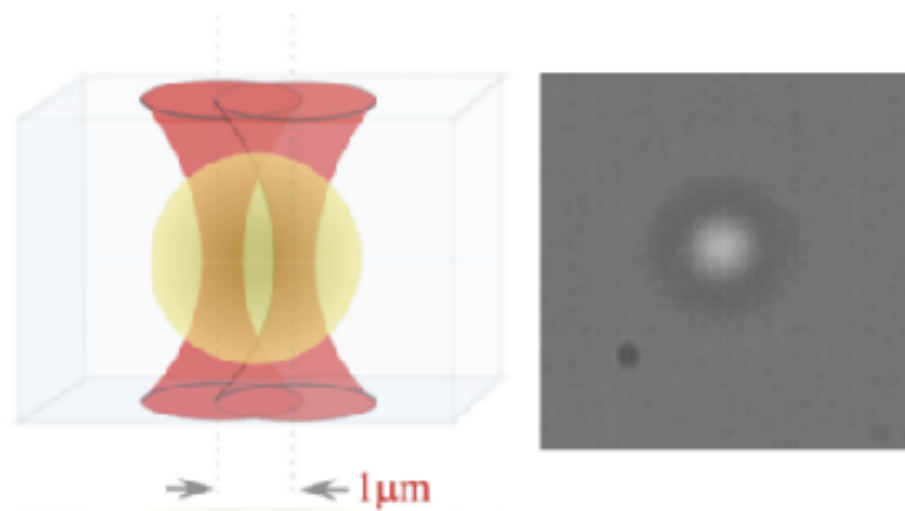


Figure 3: Energy per logic operation

# Landauer principle experimental verification

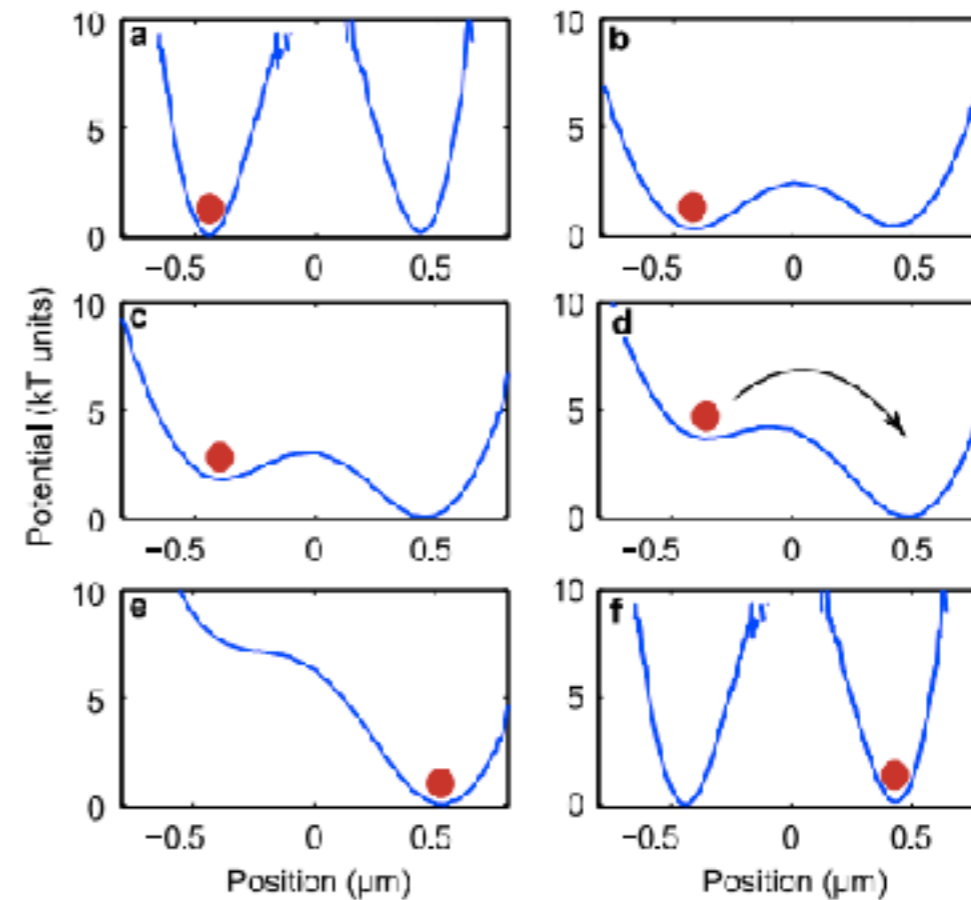
## Brownian particle in a double-well potential

Berut et al. Nature 2012



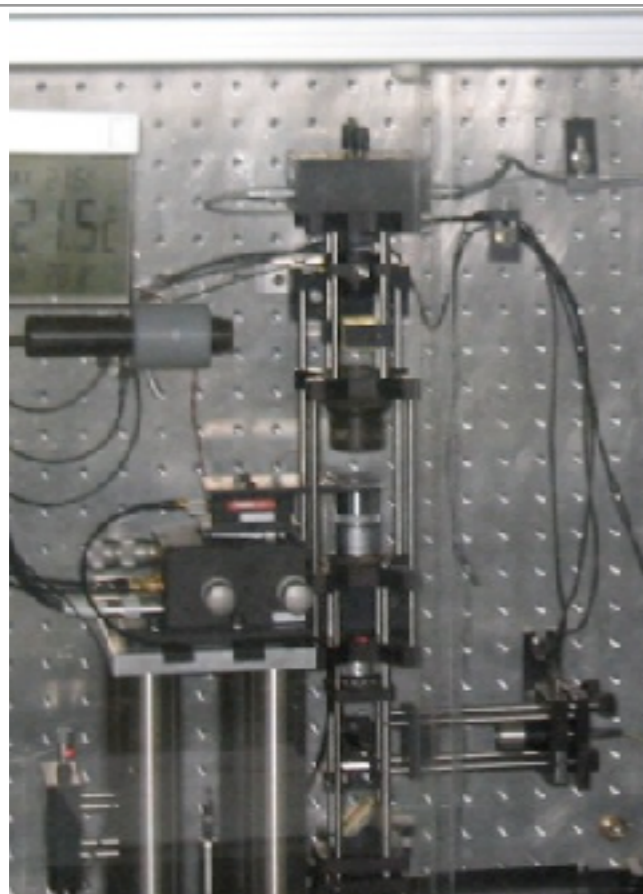
Gilberto ENS Lyon

Measured erasure cycle:

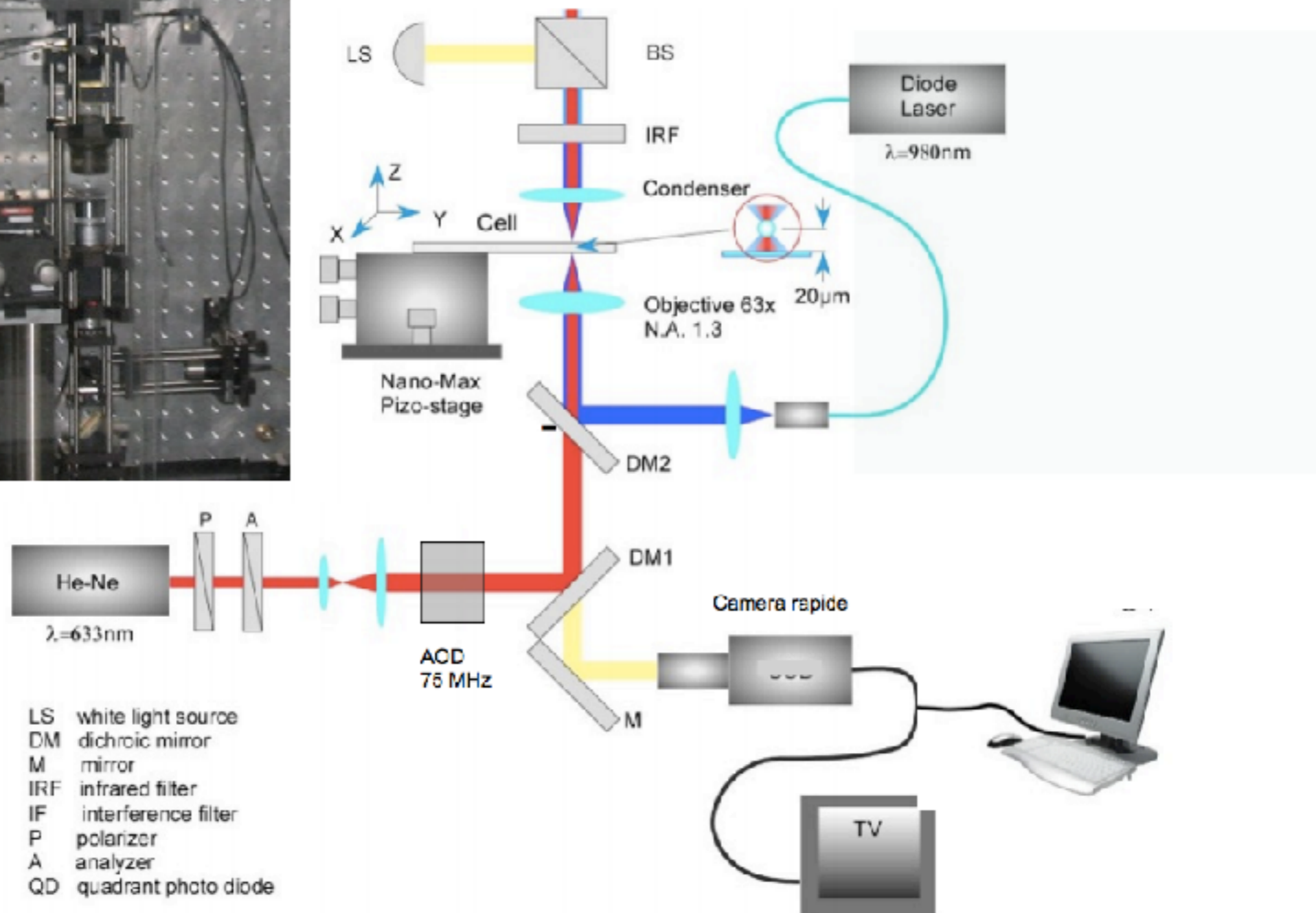


Landauer's original thought experiment

# Landauer principle experimental verification



## Experimental set-up Optical trap



Even if you're not burning books, destroying information generates heat. - Sergio Cicliberto

# Landauer principle experimental verification

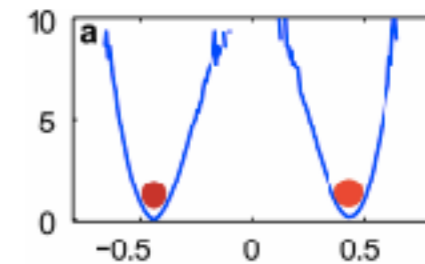
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## Generic two-state memory:

- initial configuration: **two states** with equal probability **1/2**

→ system can store **1 bit** of information

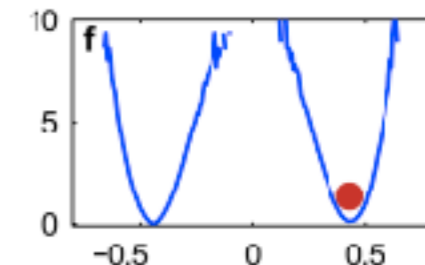
Shannon entropy:  $S_i = -\sum_n p_n \ln p_n = \ln 2$



- final configuration: **one state** with probability **1**

→ system can store **0 bit** of information

Shannon entropy:  $S_f = -\sum_n p_n \ln p_n = 0$



→ **original bit has been deleted:  $\Delta S = -\ln 2$**

# Landauer principle experimental verification

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Second law of thermodynamics (for system and reservoir):

$$\Delta S = \Delta S_{sys} + \Delta S_{res} \geq 0$$

Reservoir always in equilibrium:  $Q_{res} = T \Delta S_{res} \geq -T \Delta S_{sys}$

Equivalence between entropies:  $\Delta S_{sys} = k \Delta S = -k \ln 2$

Heat produced in reservoir:

$$Q_{res} \geq kT \ln 2$$

→ connection between information theory and thermodynamics

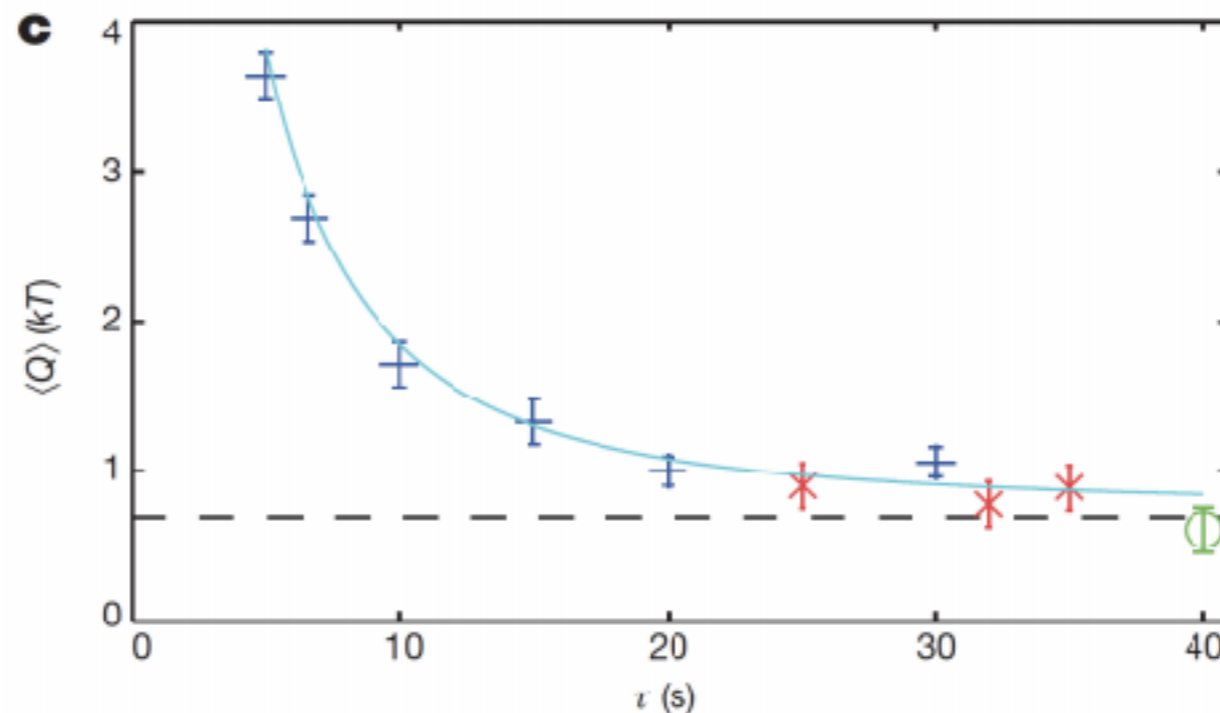
( $Q_{res} = kT \ln 2$  in quasistatic limit i.e. long cycle duration)



# Landauer principle experimental verification

## Experimental results:

We measure **work  $W$**  and deduce **heat  $Q = -\Delta U + W = W$**

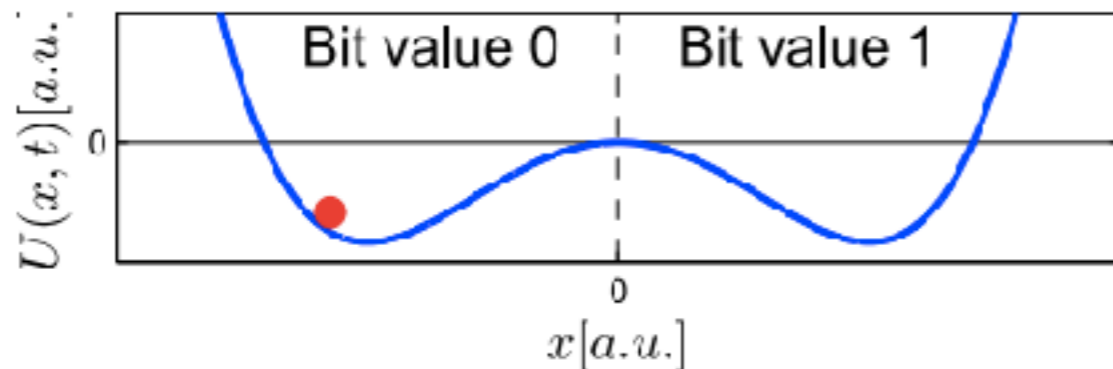


→ Landauer can be bound approached but not exceeded

Note:  $kT \ln 2 \simeq 3 \times 10^{-21} \text{ J}$  at room temperature

# Reset on colloidal particles

Colloidal particle bit



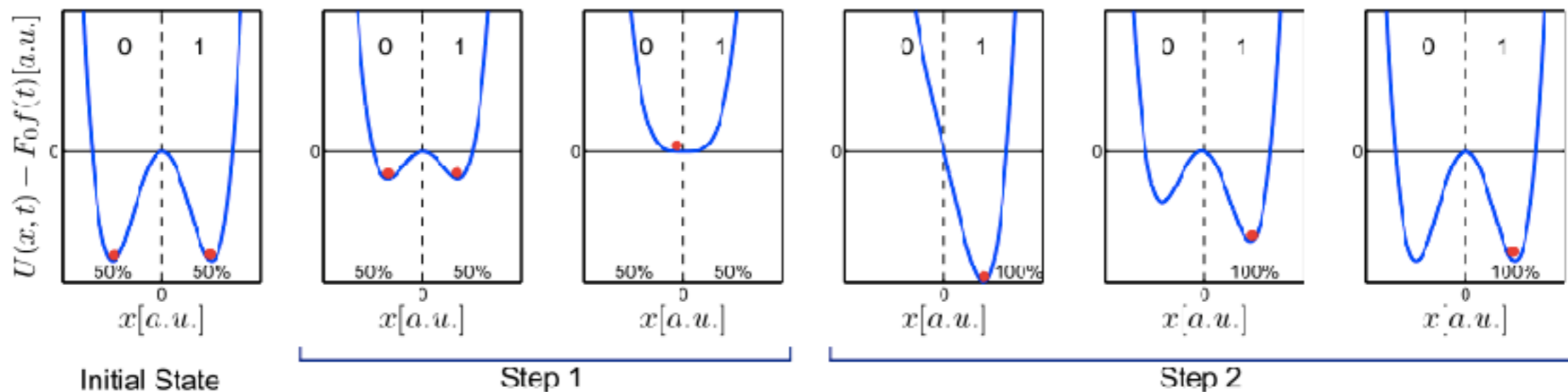
Total energy landscape

$$E(x, t) = U(x, t) - F_0 f(t)x$$

$$U(x, t) = -\frac{a}{2}g(t)x^2 + \frac{b}{4}x^4$$

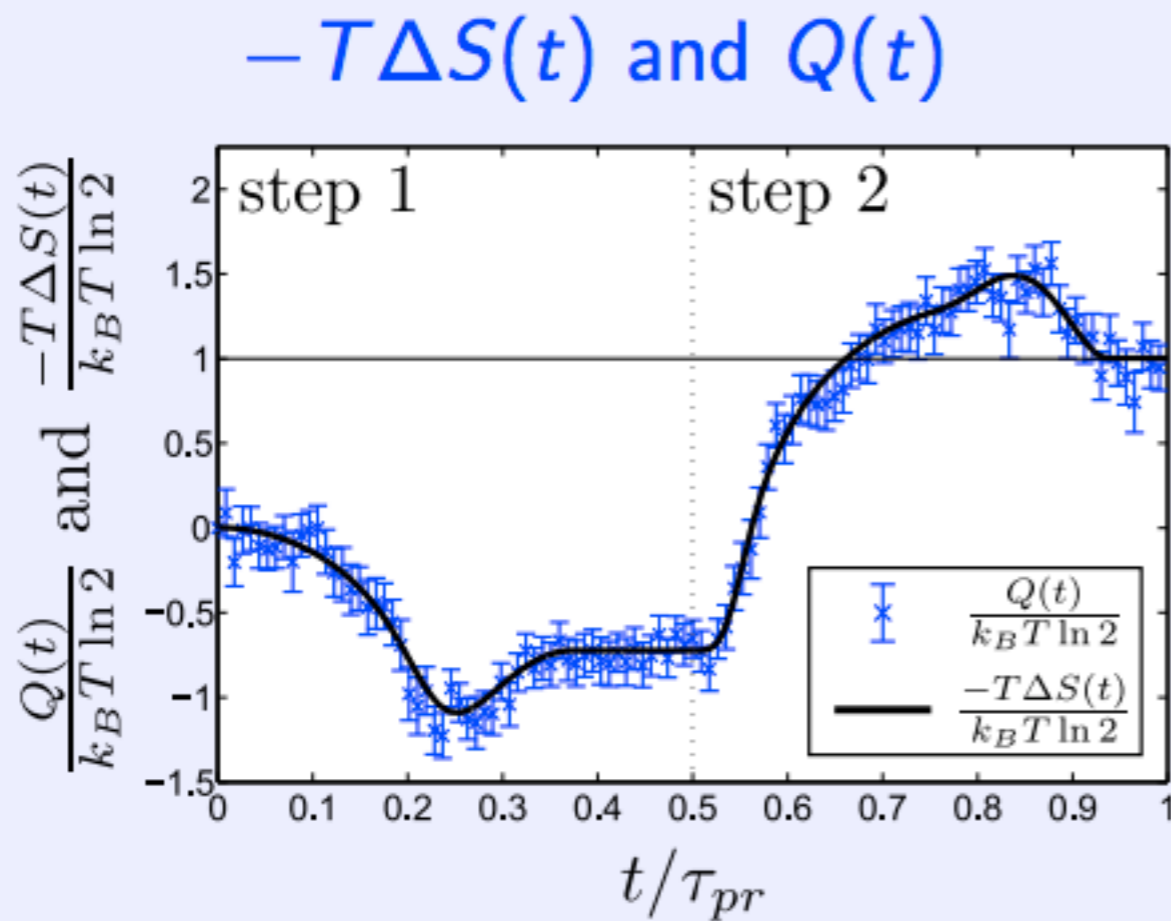
- $g(t)$  and  $f(t)$ : dimensionless parameter in  $[0, 1]$ . Their value at time  $t$  depends on a given protocol.

Reset Protocol

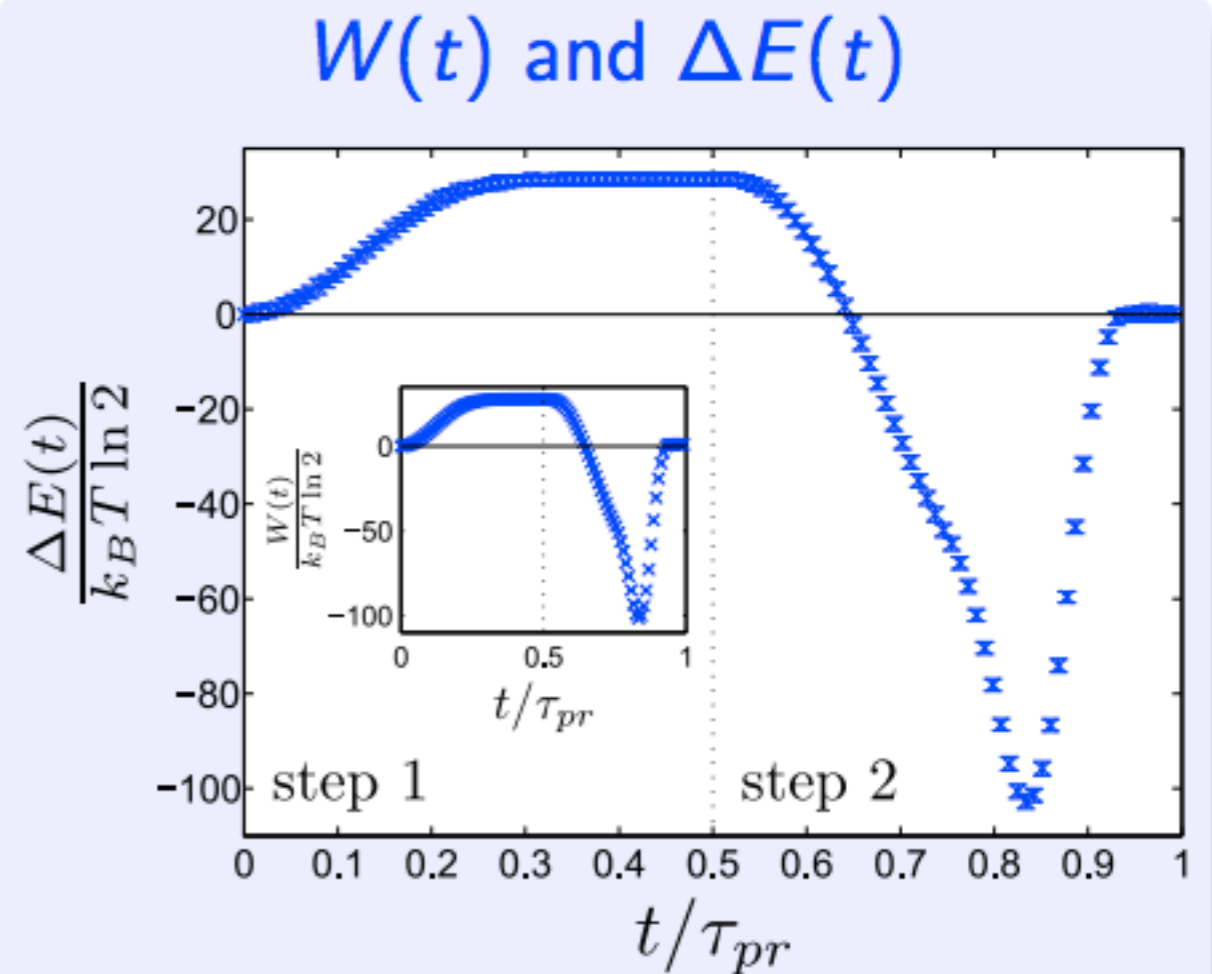


# Time-dependent study

For a fixed  $\tau_{pr}$  with  $Q(\tau_{pr}) \approx -T\Delta S(\tau_{pr})$ , study  $-T\Delta S(t)$ ,  $Q(t)$ ,  $W(t)$ ,  $\Delta E(t)$ .



$$Q(t) = -T\Delta S(t), \quad \forall t \in [0, \tau_{pr}]$$



Consistent with qualitative analysis.

# Landauer principle experimental verification

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PRL **113**, 190601 (2014)

PHYSICAL REVIEW LETTERS

week ending  
7 NOVEMBER 2014



## High-Precision Test of Landauer's Principle in a Feedback Trap

Yonggun Jun,<sup>\*</sup> Momčilo Gavrilov, and John Bechhoefer<sup>†</sup>

*Department of Physics, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada*

(Received 15 August 2014; published 4 November 2014)

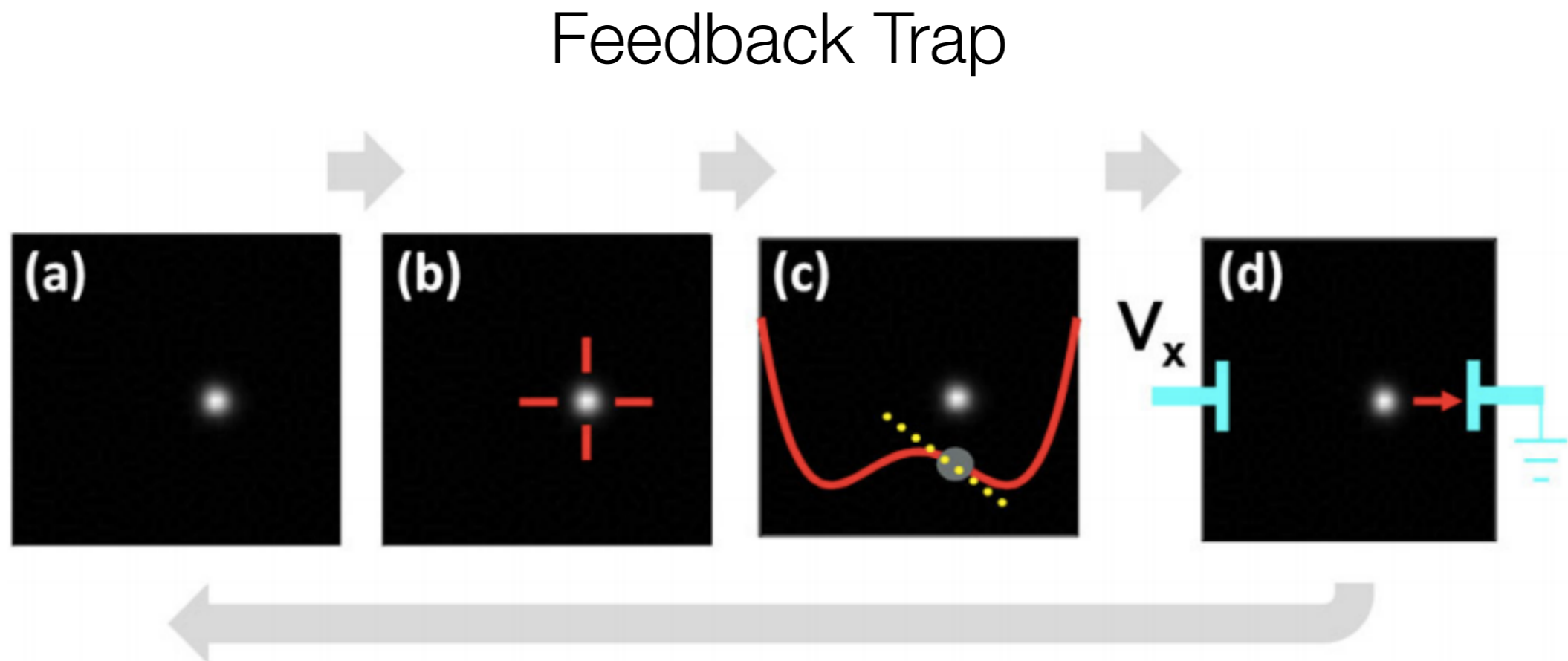
We confirm Landauer's 1961 hypothesis that reducing the number of possible macroscopic states in a system by a factor of 2 requires work of at least  $kT \ln 2$ . Our experiment uses a colloidal particle in a time-dependent, virtual potential created by a feedback trap to implement Landauer's erasure operation. In a control experiment, similar manipulations that do not reduce the number of system states can be done reversibly. Erasing information thus requires work. In individual cycles, the work to erase can be below the Landauer limit, consistent with the Jarzynski equality.

DOI: [10.1103/PhysRevLett.113.190601](https://doi.org/10.1103/PhysRevLett.113.190601)

PACS numbers: 05.70.Ln, 03.67.-a, 05.20.-y, 05.90.+m

# Landauer principle experimental verification

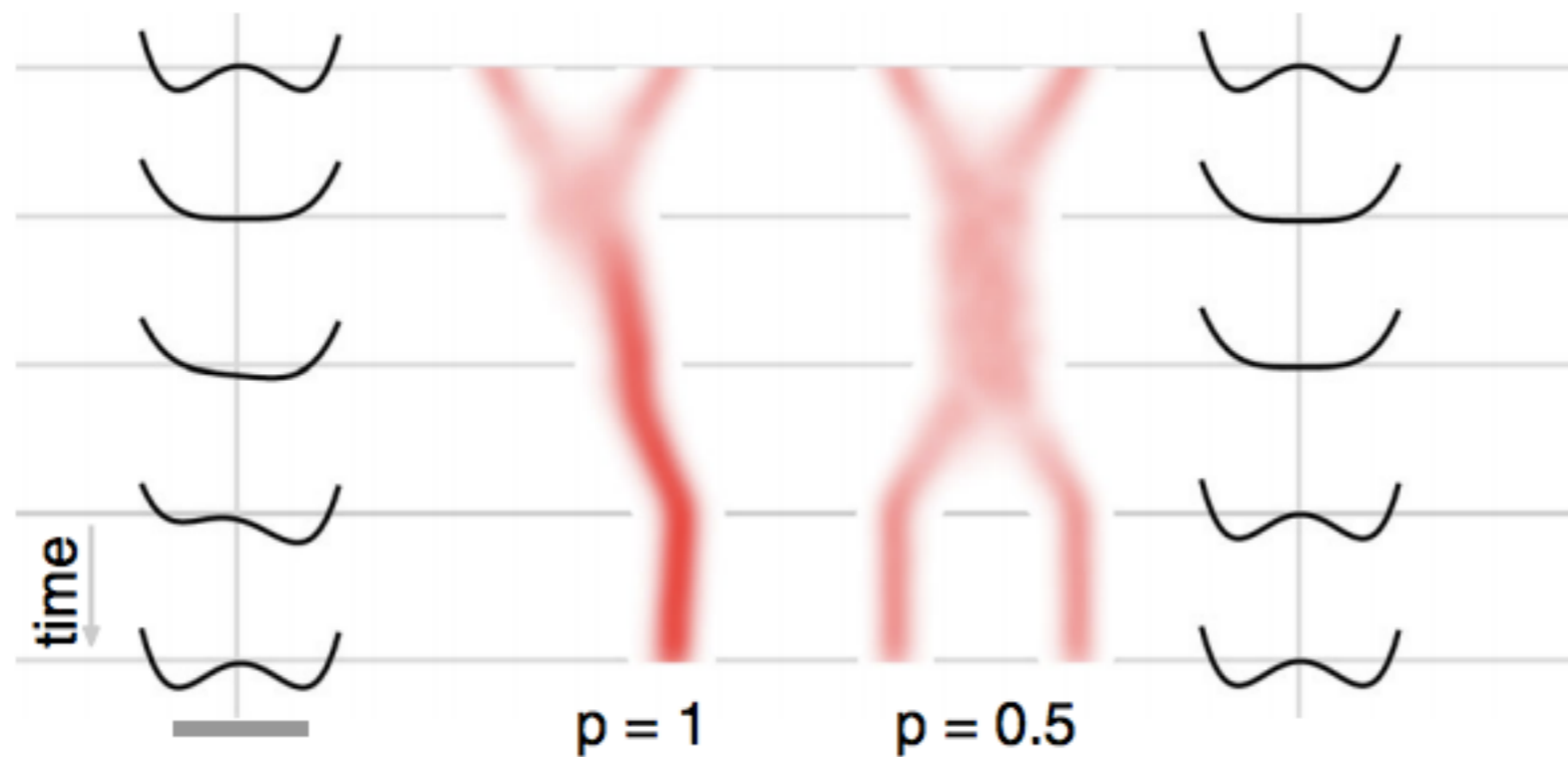
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# Landauer principle experimental verification

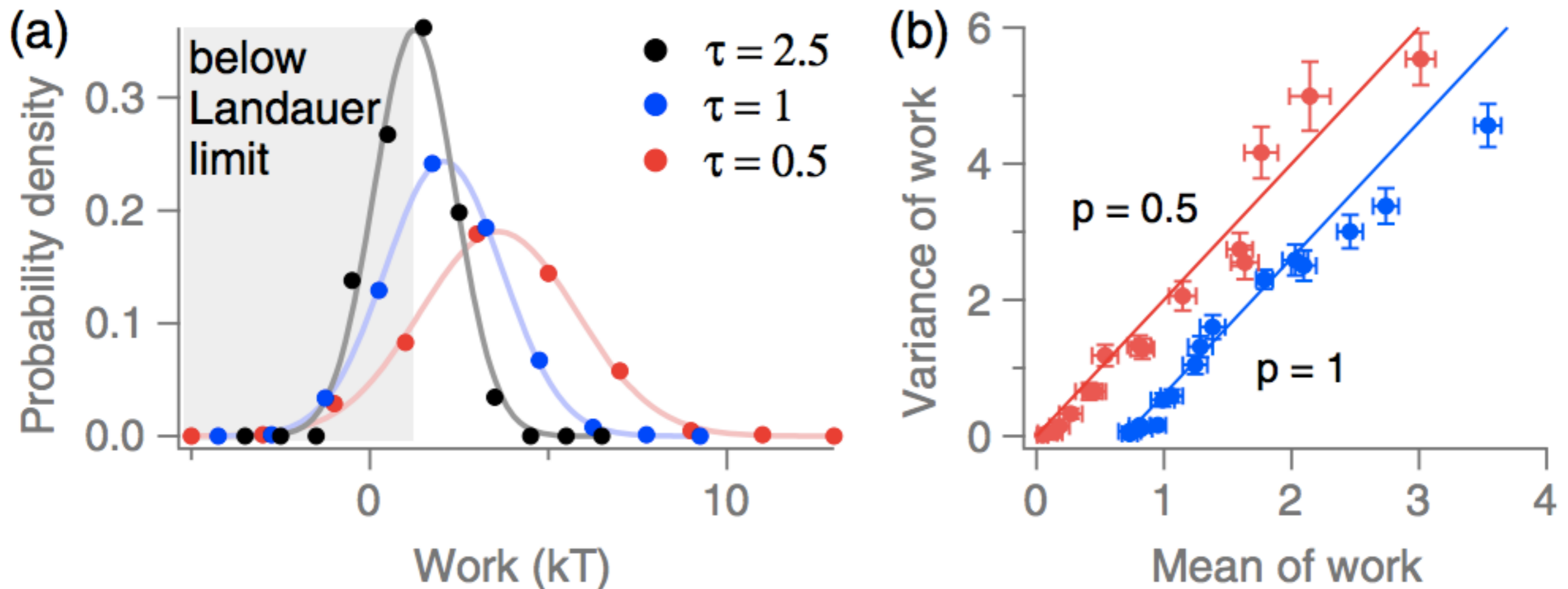
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## Erasure protocol

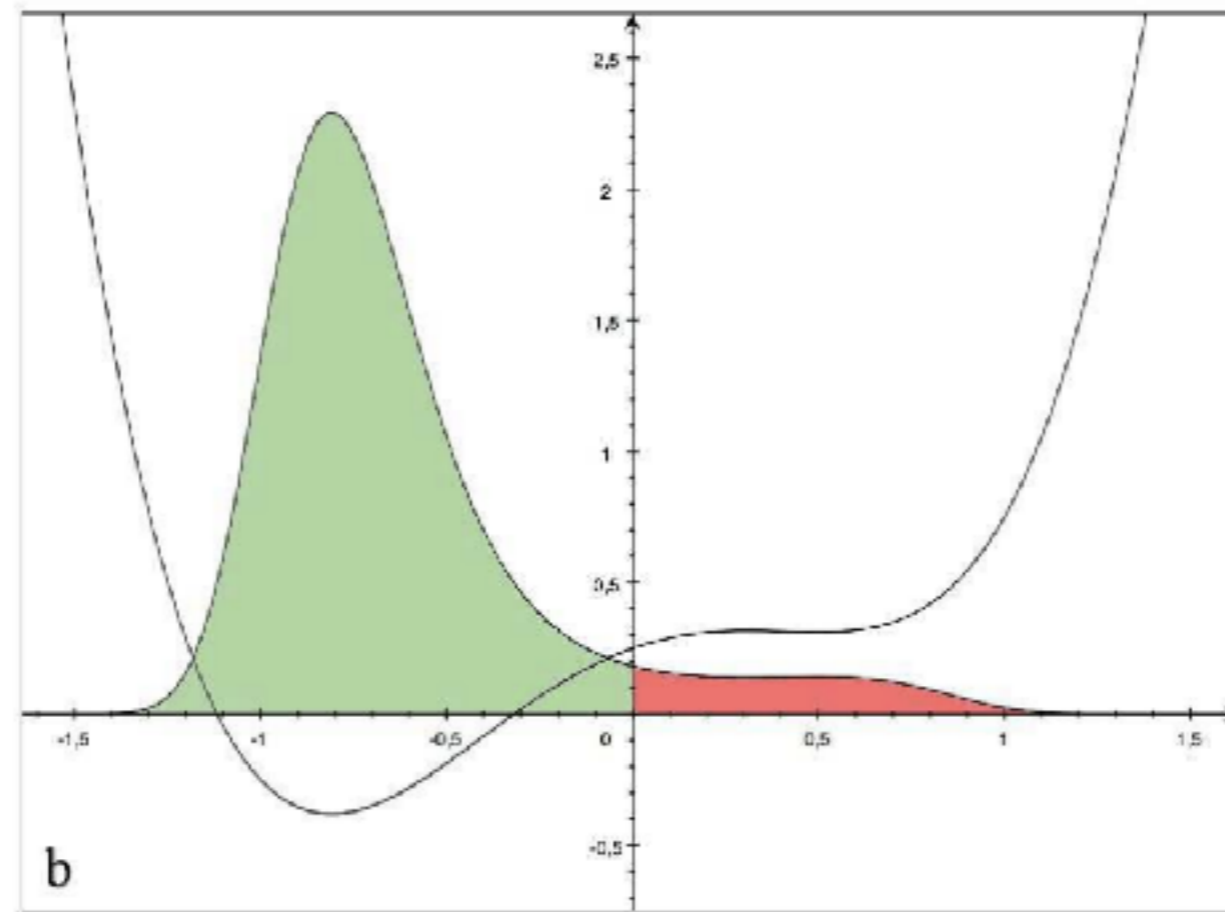


# Landauer principle experimental verification

Work series for individual cycles



# Beating the Landauer's limit by trading energy with uncertainty



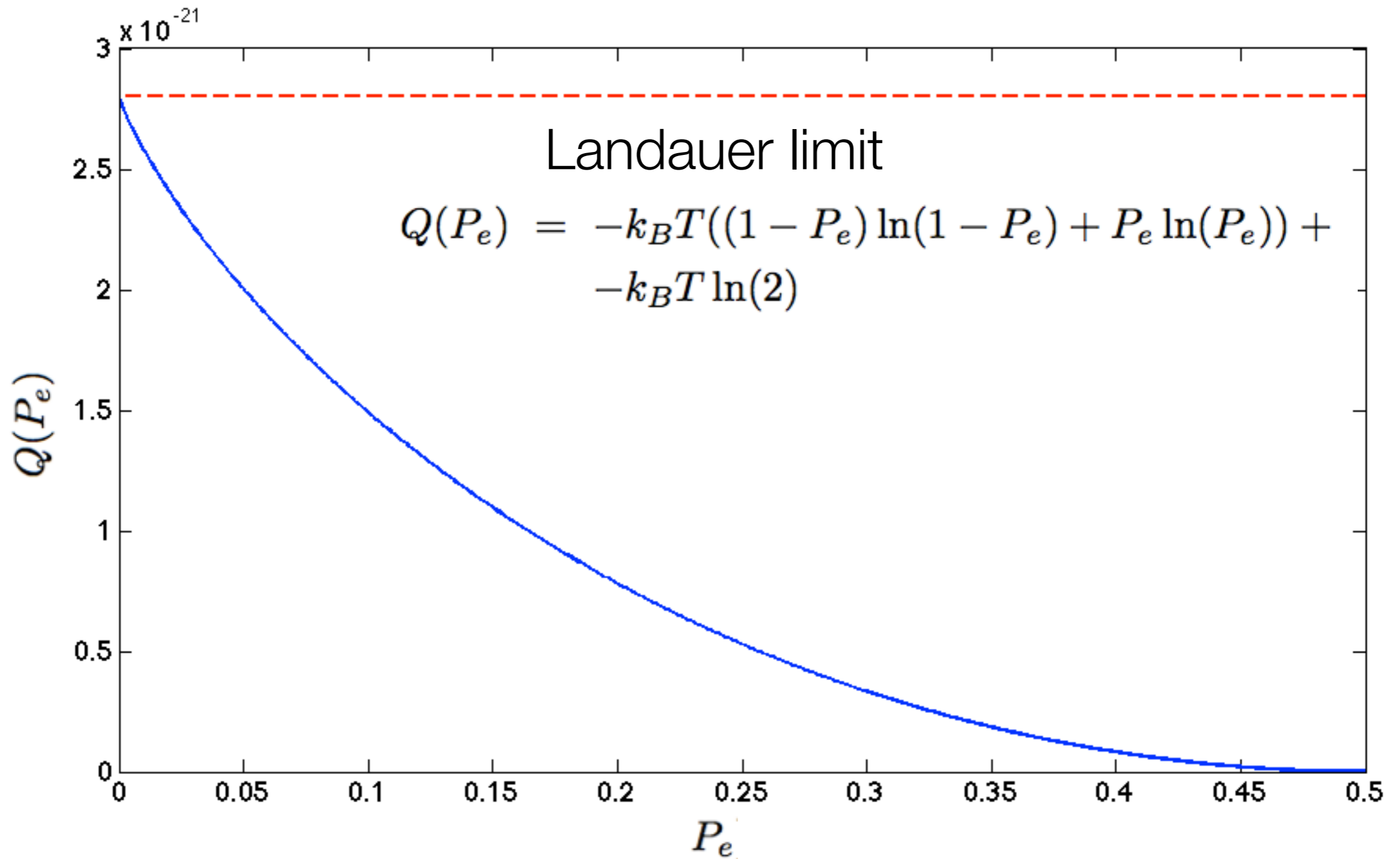
$$\Delta S = S_f - S_i = k_B (\ln(1) - \ln(2)) = -k_B \ln(2)$$

$$S_f(P_e) = -k_B ((1 - P_e) \ln(1 - P_e) + P_e \ln(P_e))$$

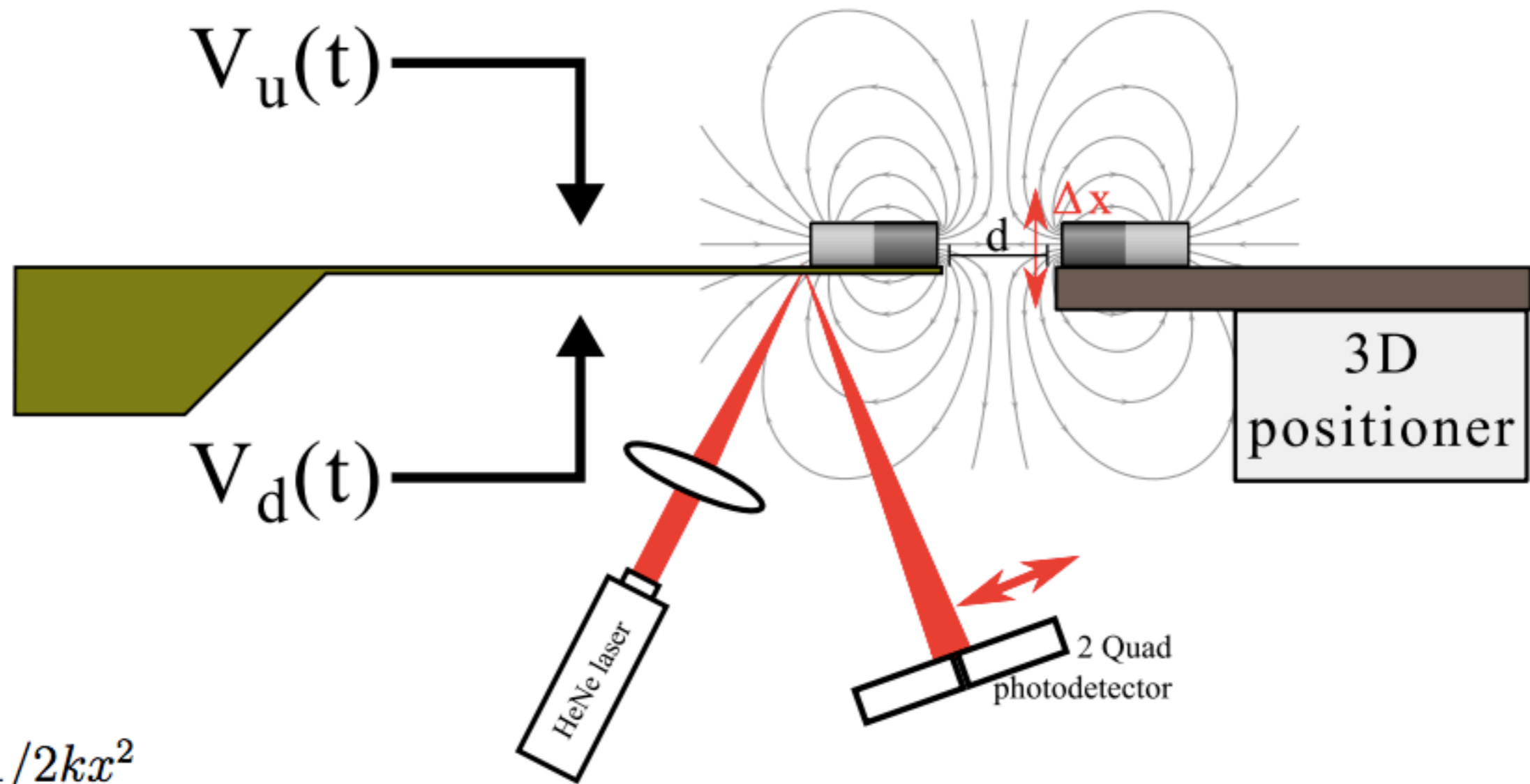
$$Q(P_e) = -k_B T ((1 - P_e) \ln(1 - P_e) + P_e \ln(P_e)) + \\ -k_B T \ln(2)$$



# Beating the Landauer's limit by trading energy with uncertainty



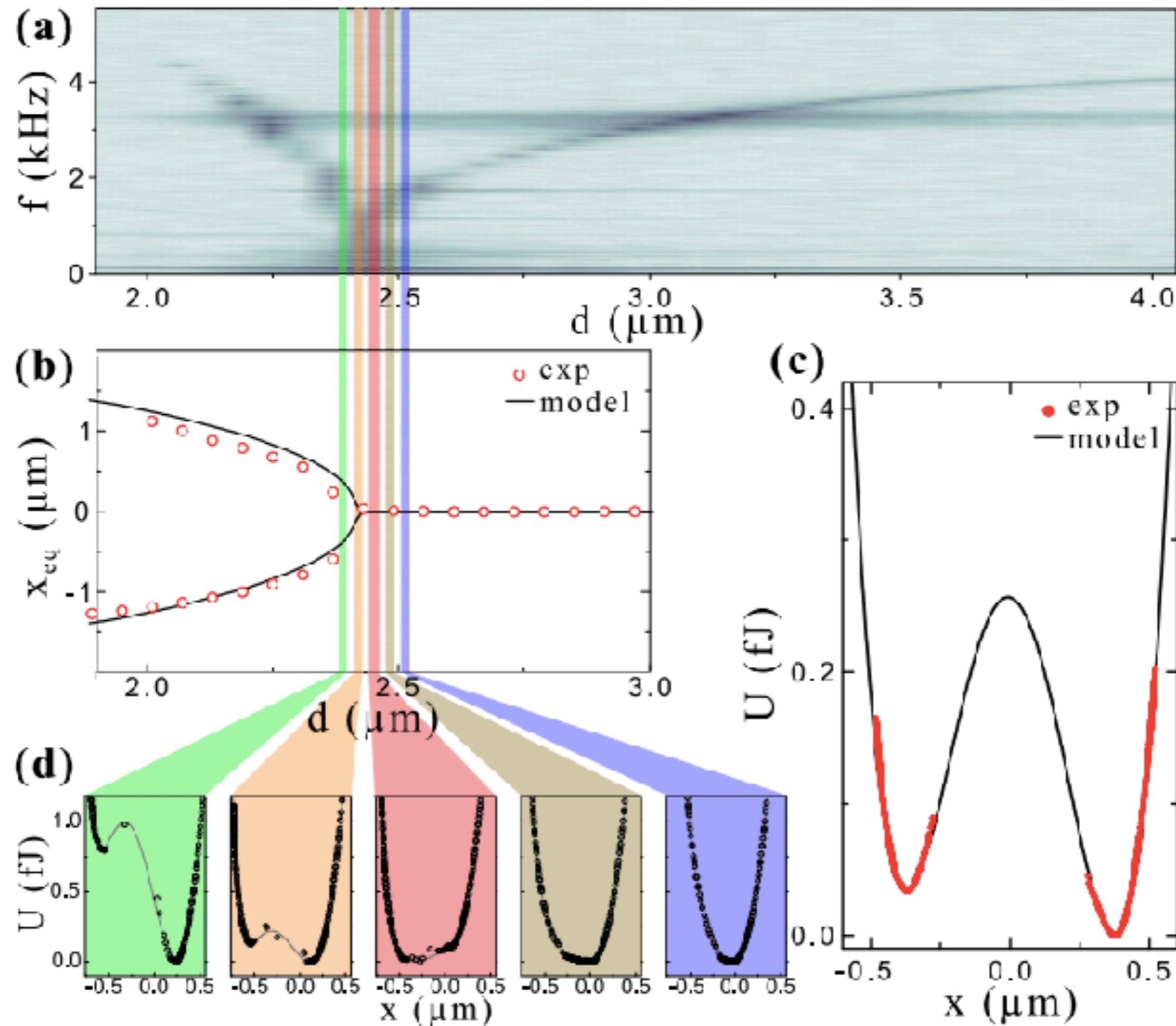
# Micro-electromechanical memory bit based on magnetic repulsion



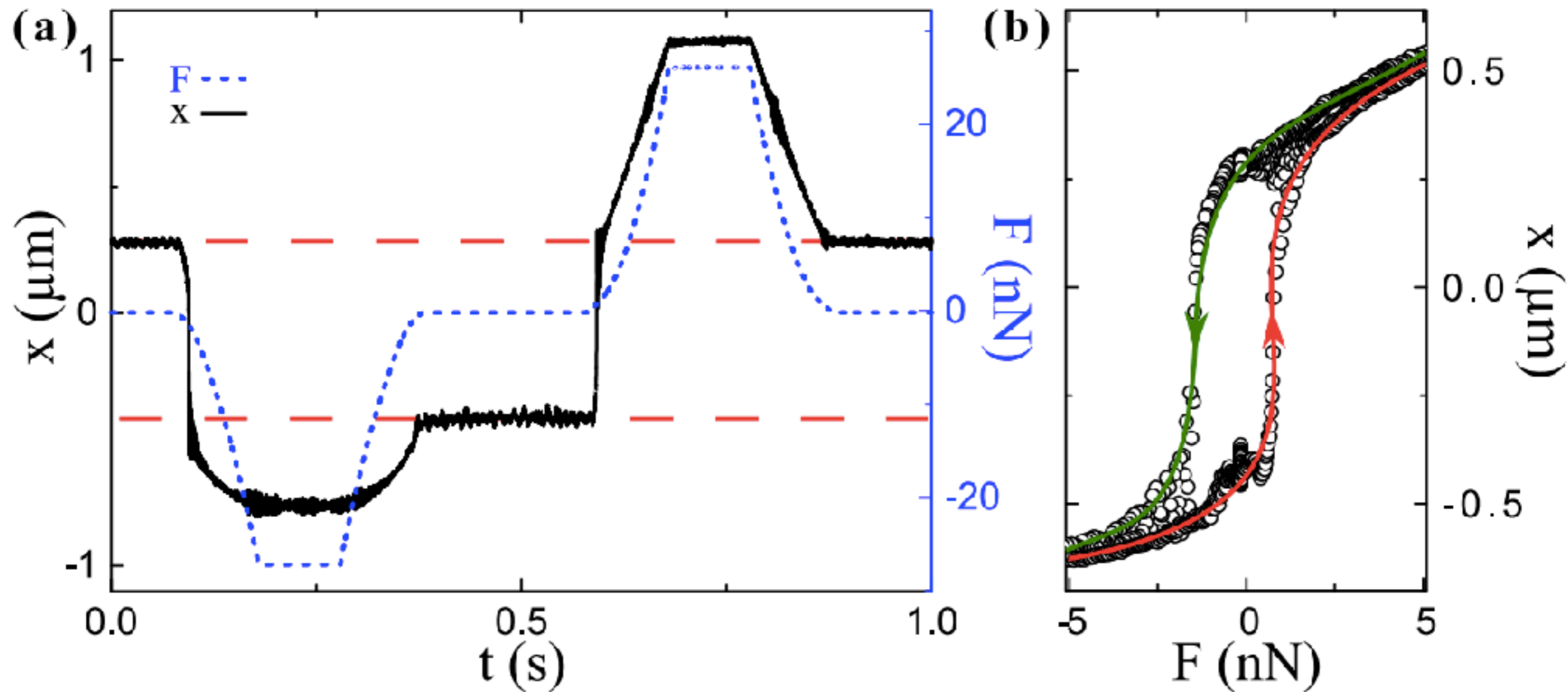
$$U_{el} = 1/2kx^2$$

$$\mathbf{F}_m(\mathbf{r}, \mathbf{m}_1, \mathbf{m}_2) = \frac{3\mu_0}{4\pi r^5} [(\mathbf{m}_1 \cdot \mathbf{r})\mathbf{m}_2 + (\mathbf{m}_2 \cdot \mathbf{r})\mathbf{m}_1 + (\mathbf{m}_1 \cdot \mathbf{m}_2)\mathbf{r} - \frac{5(\mathbf{m}_1 \cdot \mathbf{r})(\mathbf{m}_2 \cdot \mathbf{r})}{r^2}\mathbf{r}]$$

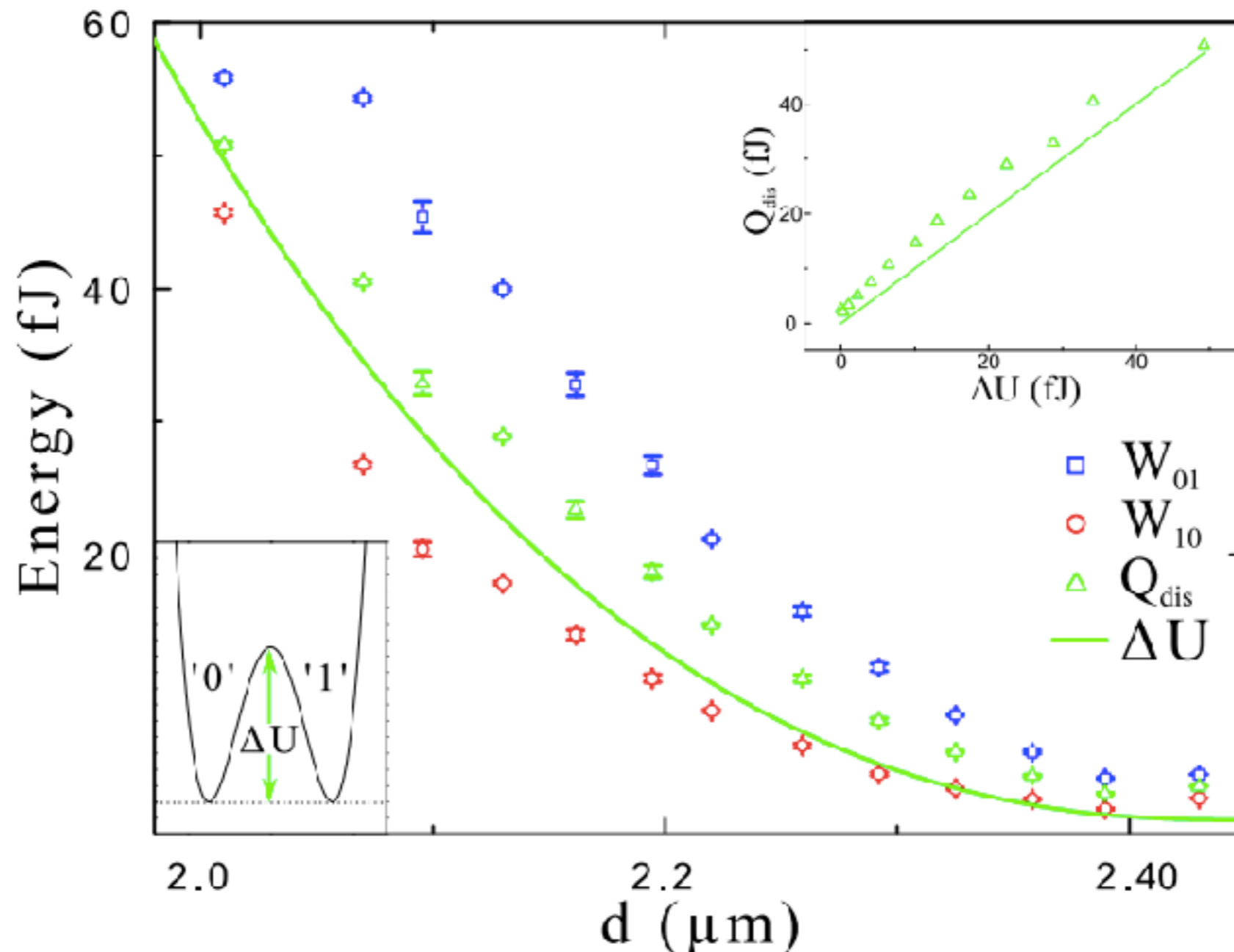
# Micro-electromechanical memory bit based on magnetic repulsion



# Micro-electromechanical memory bit based on magnetic repulsion

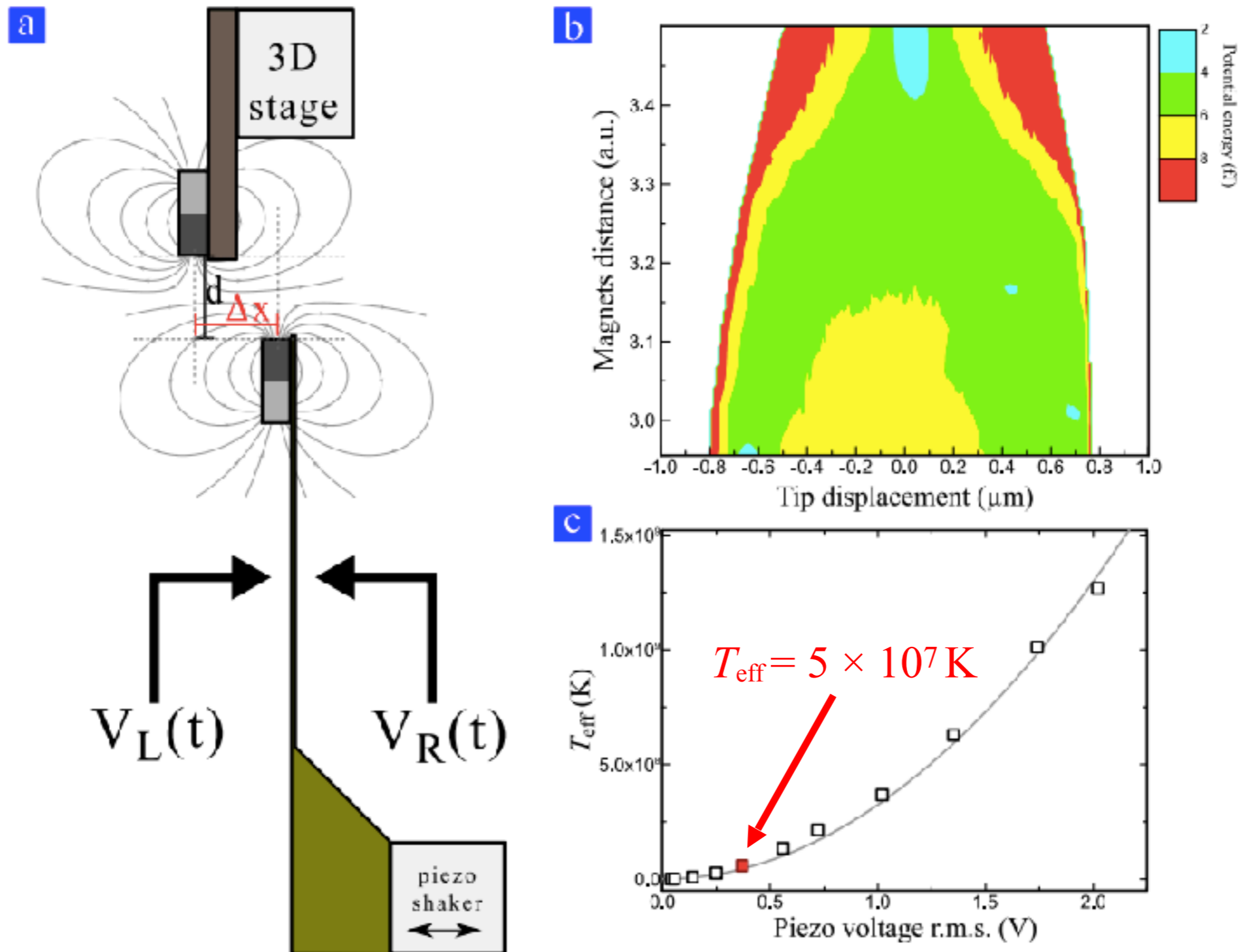


# Micro-electromechanical memory bit based on magnetic repulsion

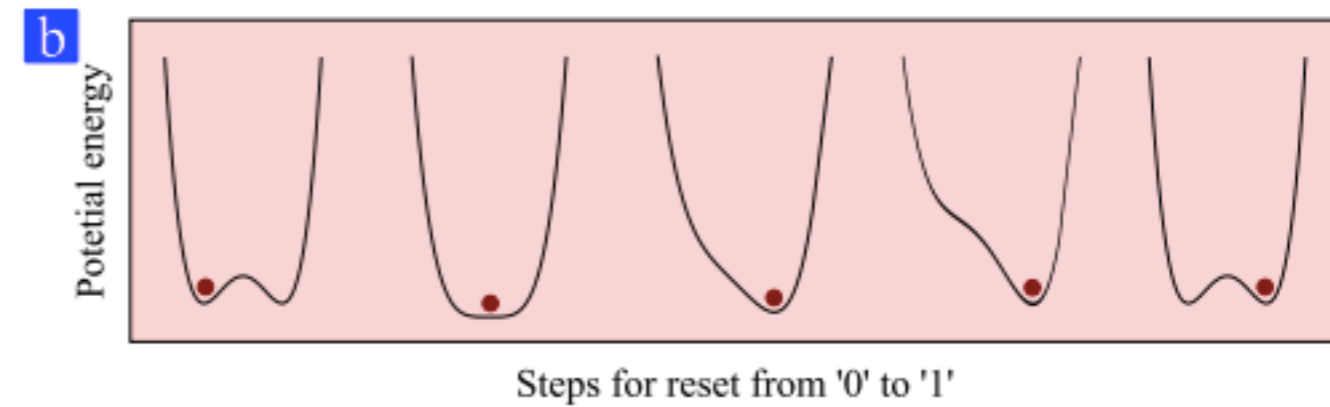
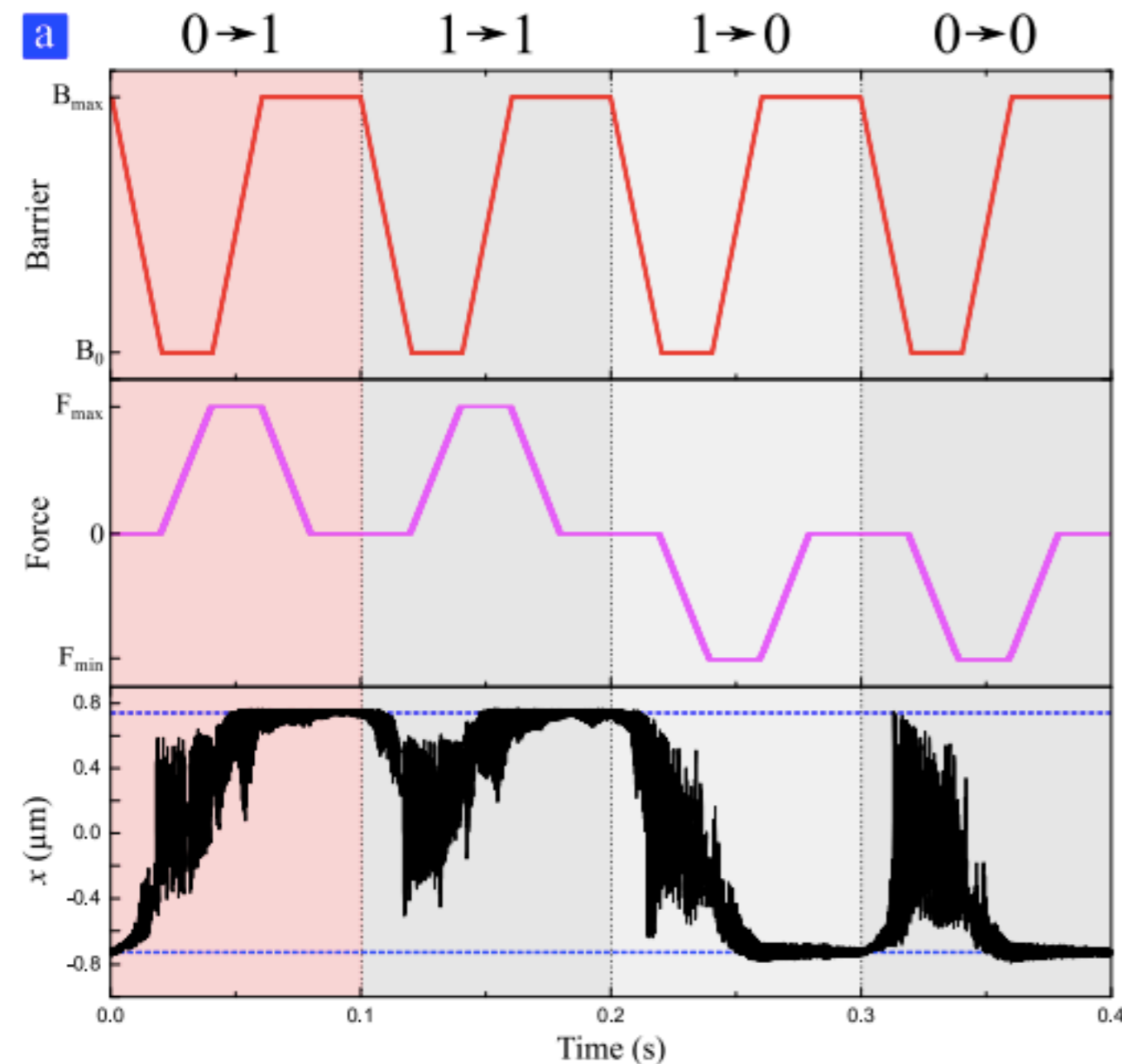


Orders of magnitude above Landauer limit!

# Solution: increase the temperature



# Reset protocol

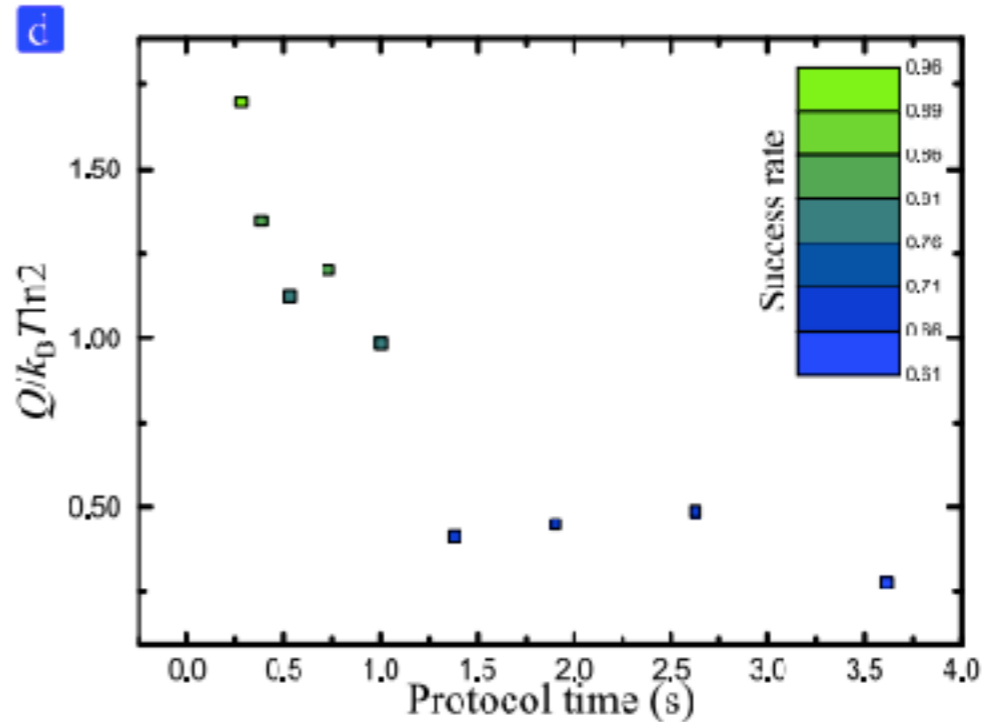
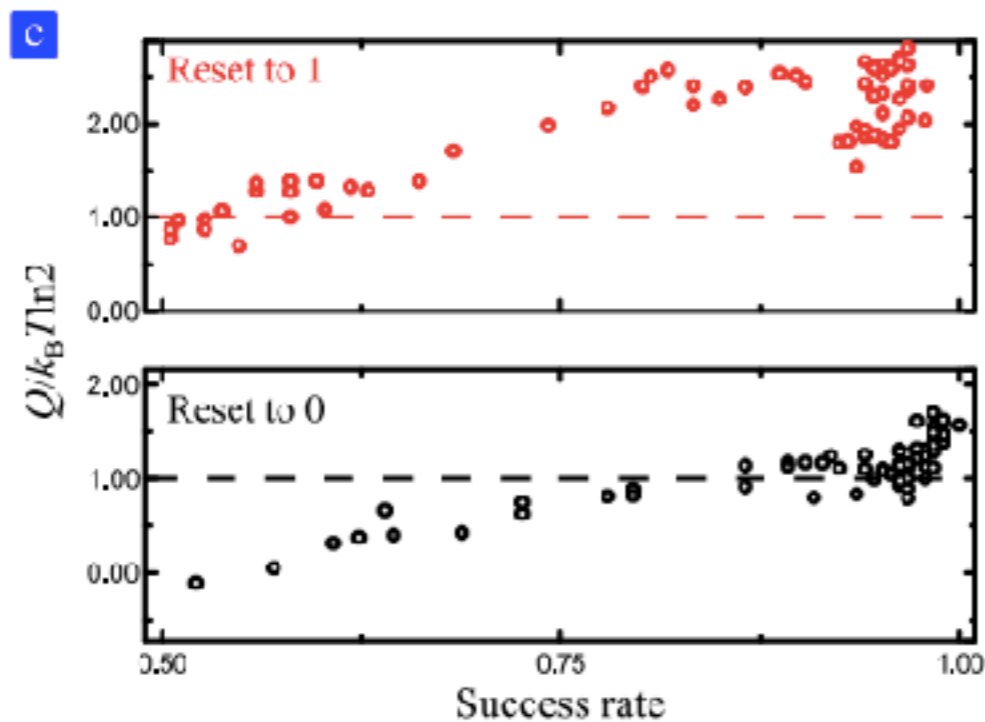
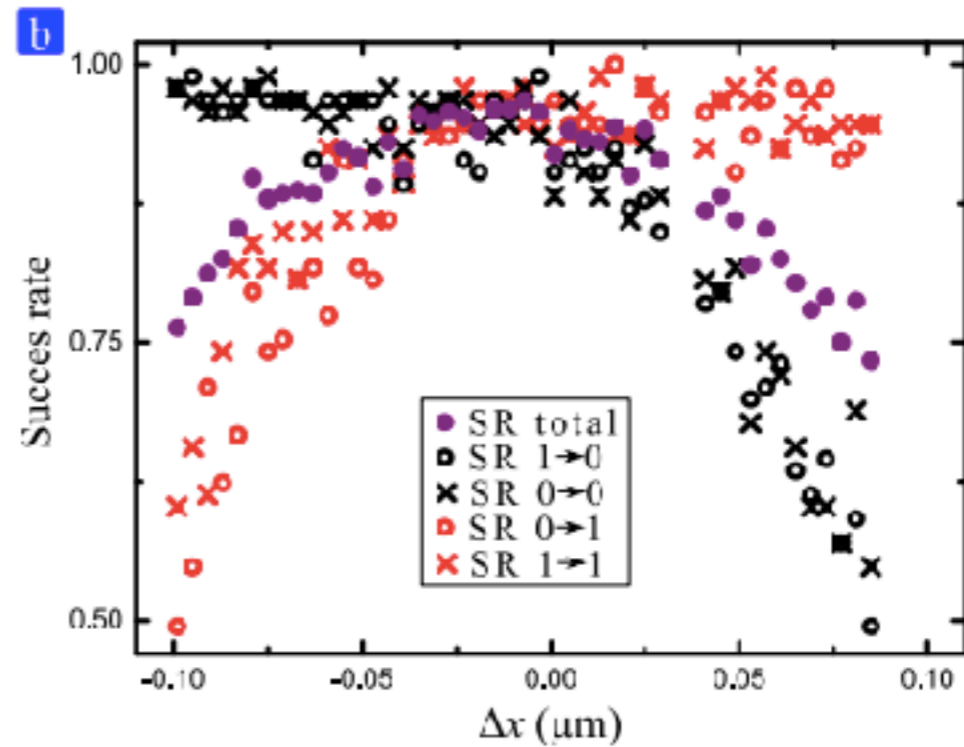
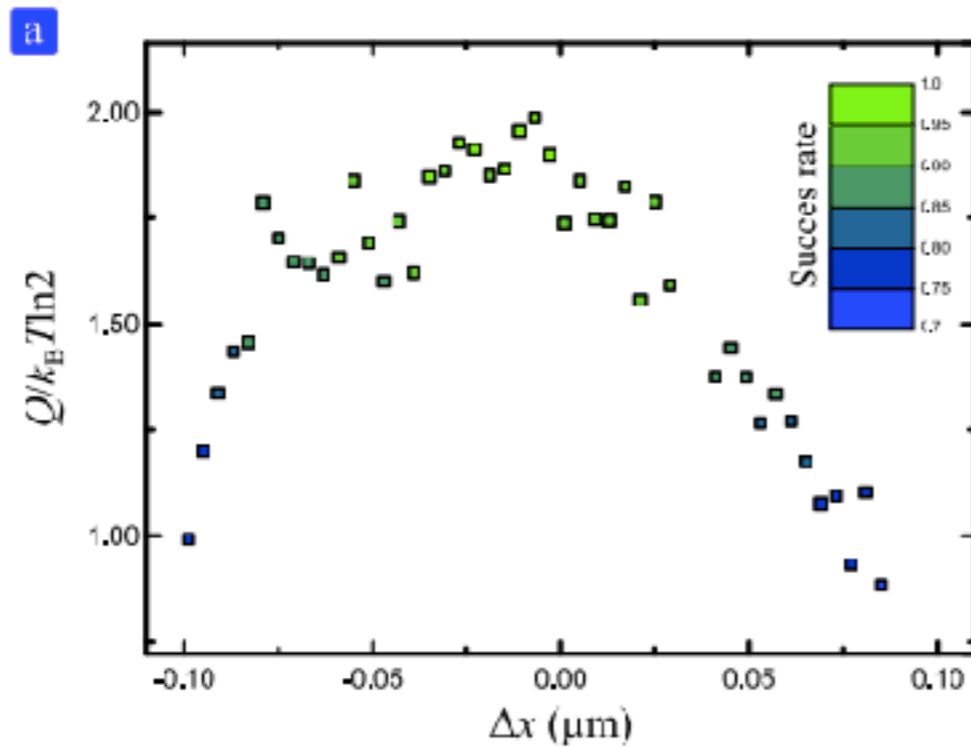


$$W = \int_0^{\tau_p} \sum_{k=1}^M \frac{\partial U(x, \boldsymbol{\lambda})}{\partial \lambda_k} \frac{\partial \lambda_k}{\partial t} dt$$

$$Q = W - \Delta U$$

$$Q(P_s) \geq k_B T [\ln(2) + P_s \ln(P_s) + (1 - P_s) \ln(1 - P_s)]$$

# Landauer reset with error





# Logically irreversible devices

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*We shall call a device logically irreversible if the output of a device does not uniquely define the inputs. We believe that devices exhibiting logical irreversibility are essential to computing. Logical irreversibility, we believe, in turn implies physical irreversibility, and the latter is accompanied by dissipative effects.*

# Information is Physical

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Rolf Landauer, 1961. Whenever we use a logically irreversible gate we dissipate energy into the environment.

# Logically irreversible devices

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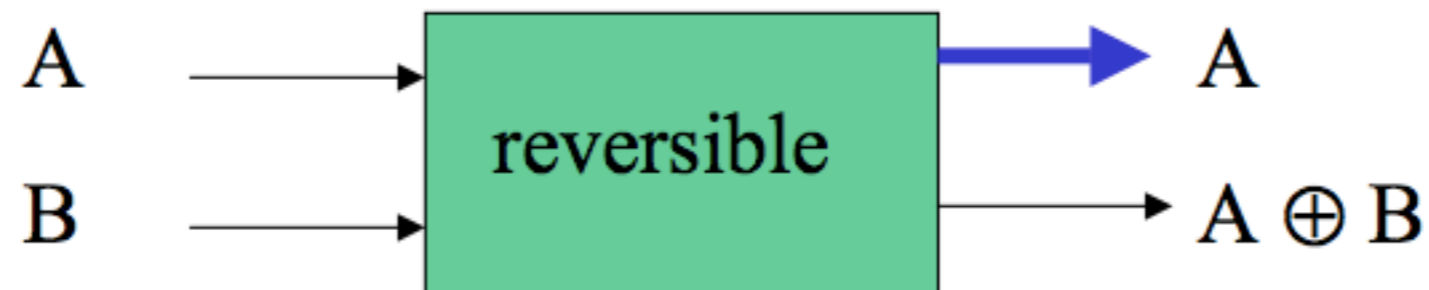
*Landauer has posed the question of whether logical irreversibility is an unavoidable feature of useful computers, arguing that it is, and has demonstrated the physical and philosophical importance of this question by showing that whenever a physical computer throws away information about its previous state it must generate a corresponding amount of entropy. Therefore, a computer must dissipate at least  $k_B T \ln 2$  of energy (about  $3 \times 10^{-21}$  Joule at room temperature) for each bit of information it erases or otherwise throws away.*



# Solution = Reversibility

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- Charles Bennett, 1973: There are no unavoidable energy consumption requirements per step in a computer.



- Energy dissipation of reversible circuit, under ideal physical circumstances, is zero.

# Reversible computation

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- **Landauer/Bennett:** all operations required in computation could be performed in a reversible manner, thus dissipating no heat.
- The first condition for any deterministic device to be reversible is that its input and output be uniquely retrievable from each other, then it is called **logically reversible**.
- The second condition: a device can actually run backwards, then it is called **physically reversible**, and the second law of thermodynamics guarantees that it dissipates no heat.

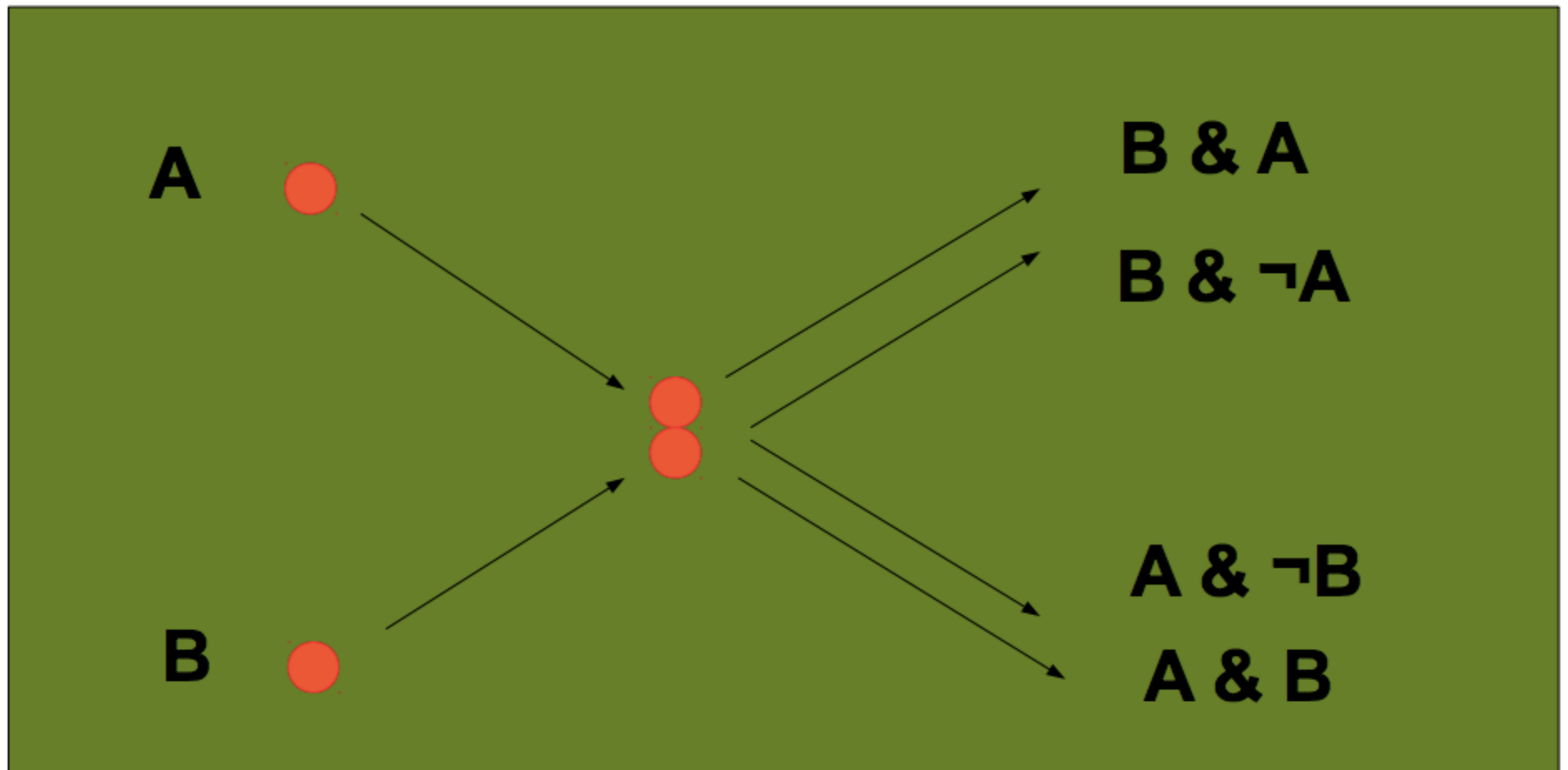
# Billiard ball computing

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- Model of a reversible mechanical computer based on Newtonian dynamics
- Proposed in 1982 by Edward Fredkin and Tommaso Toffoli
- It relies on the motion of spherical billiard balls in a friction-free environment made of buffers against which the balls bounce perfectly

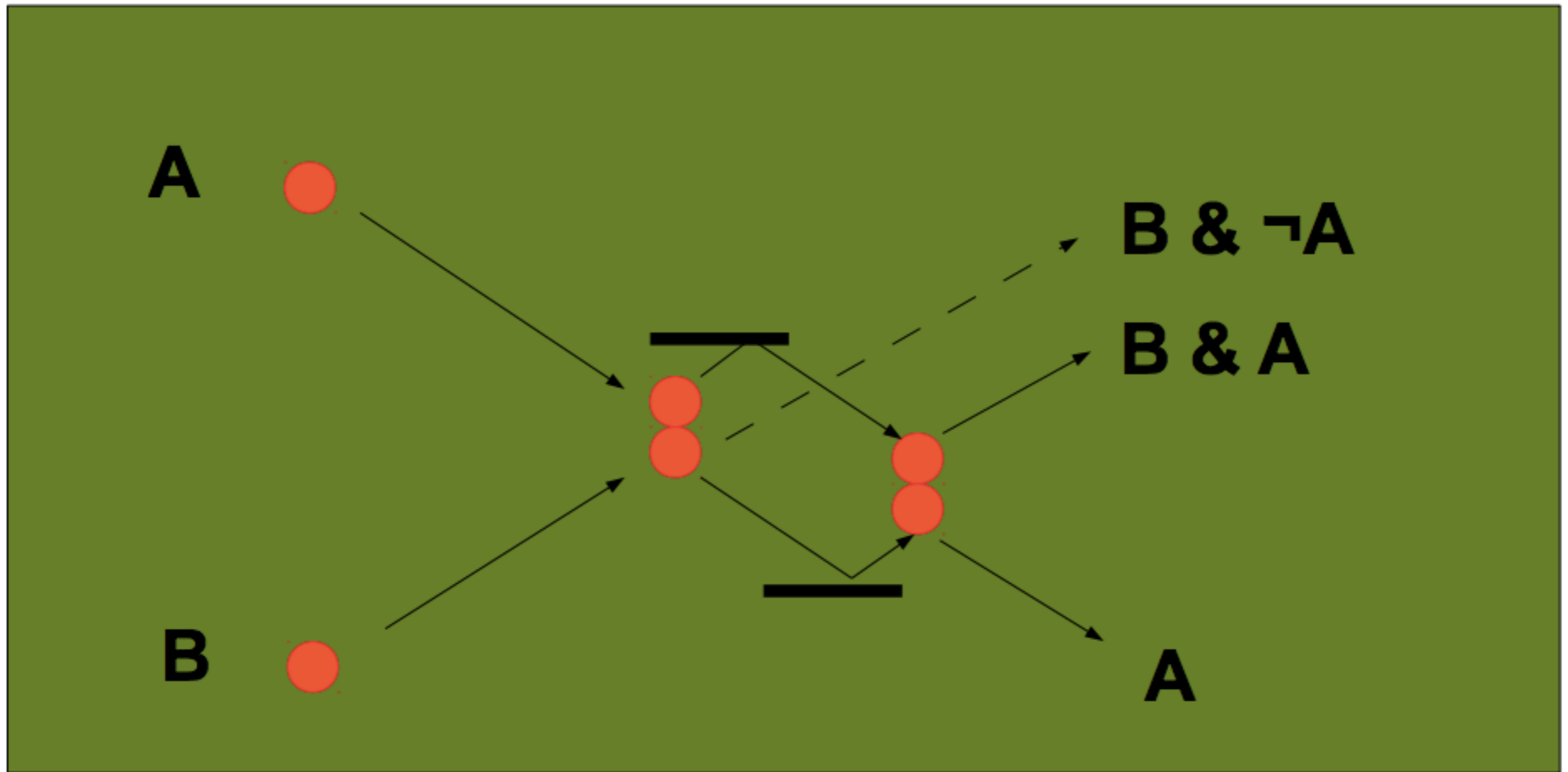
# Billiard ball computing

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Assume no friction, elastic collisions

# Billiard ball computing



Use “mirrors” to implement “switching device”  
This device is *reversible* because physics is



# Billiard ball computing

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- Using balls and mirrors, we can implement basic logic gates: AND, OR, NOT
- With a big enough billiard table, we could (in theory) implement a complete computer using a combination of these gates
- BUT...
  - billiard balls don't work in practice

# Billiard ball computing

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- **Thermal losses**

- friction can't be ignored
- Collisions aren't perfectly elastic

- **Chaotic motion**

- Balls are actually conglomerates of many atoms in various states of vibration
- Can't know their “initial state” perfectly
- Small variations in initial conditional conditions can cause exponentially large differences in final state

# Reversible computing

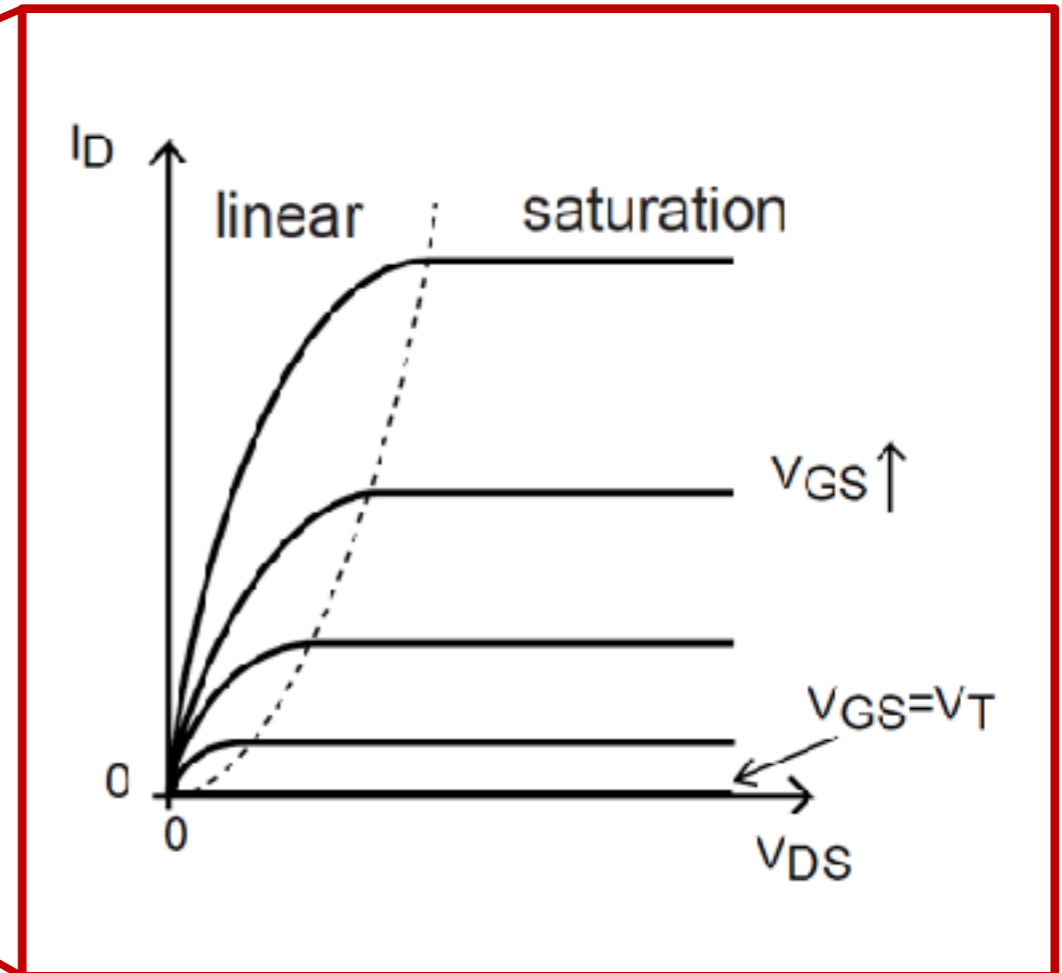
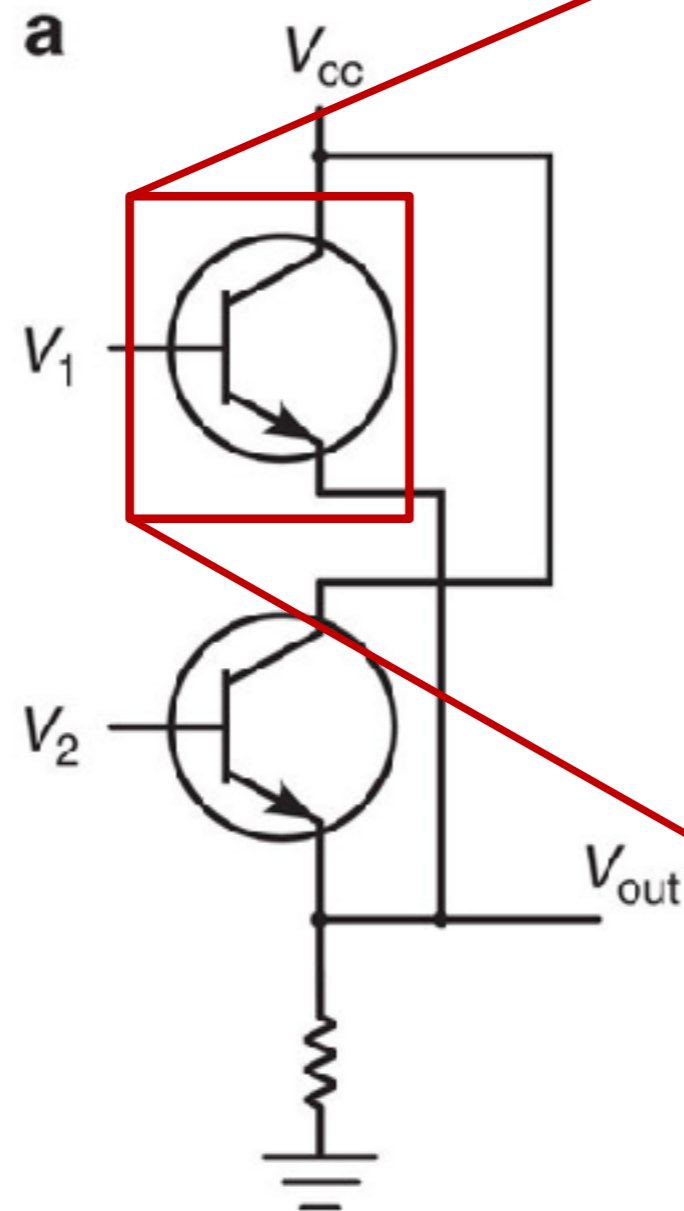
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- The reasoning on connection between physical and logical reversibility **applies only to systems that encode input and outputs** on the system itself.
- If the input and output are not part of the computing system (like in transistor based logic gates) **there is no connection between physical and logical reversibility.**

# Back to the real world....

## OR gate

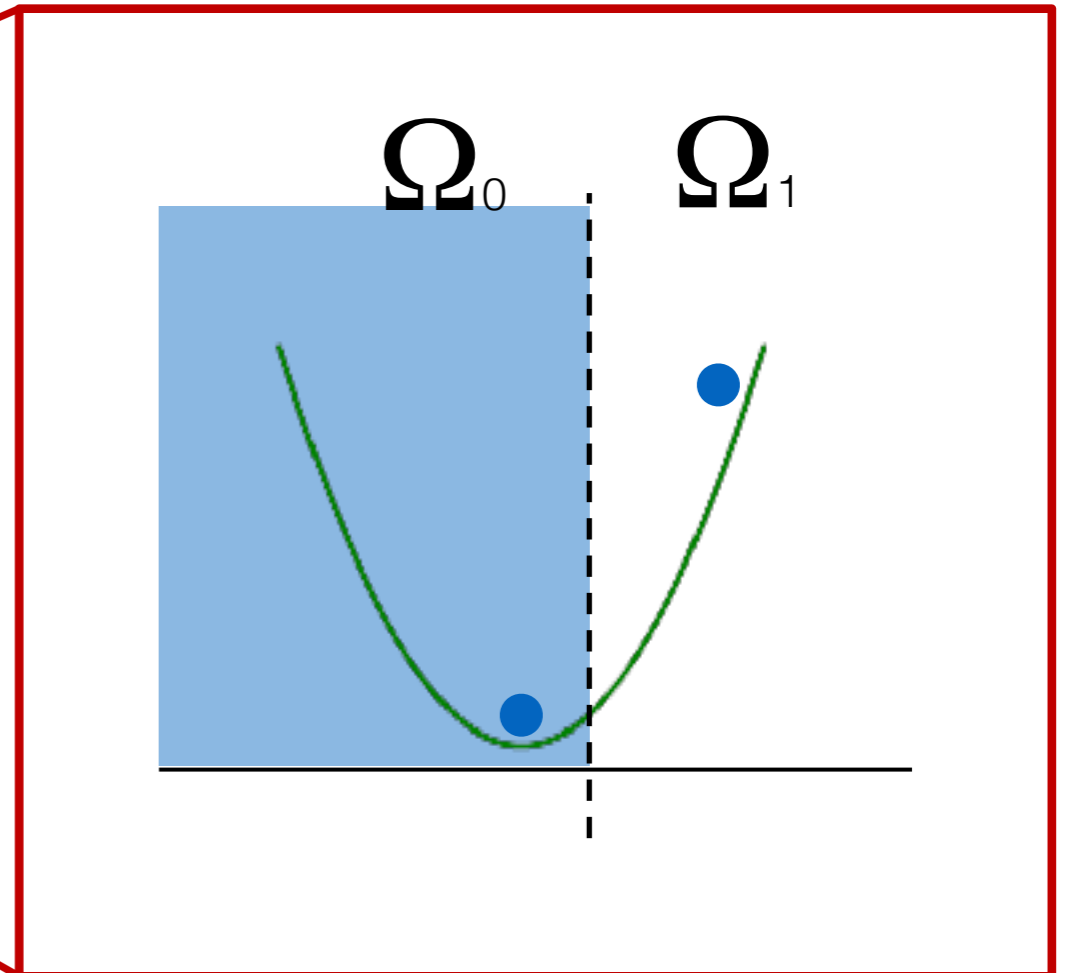
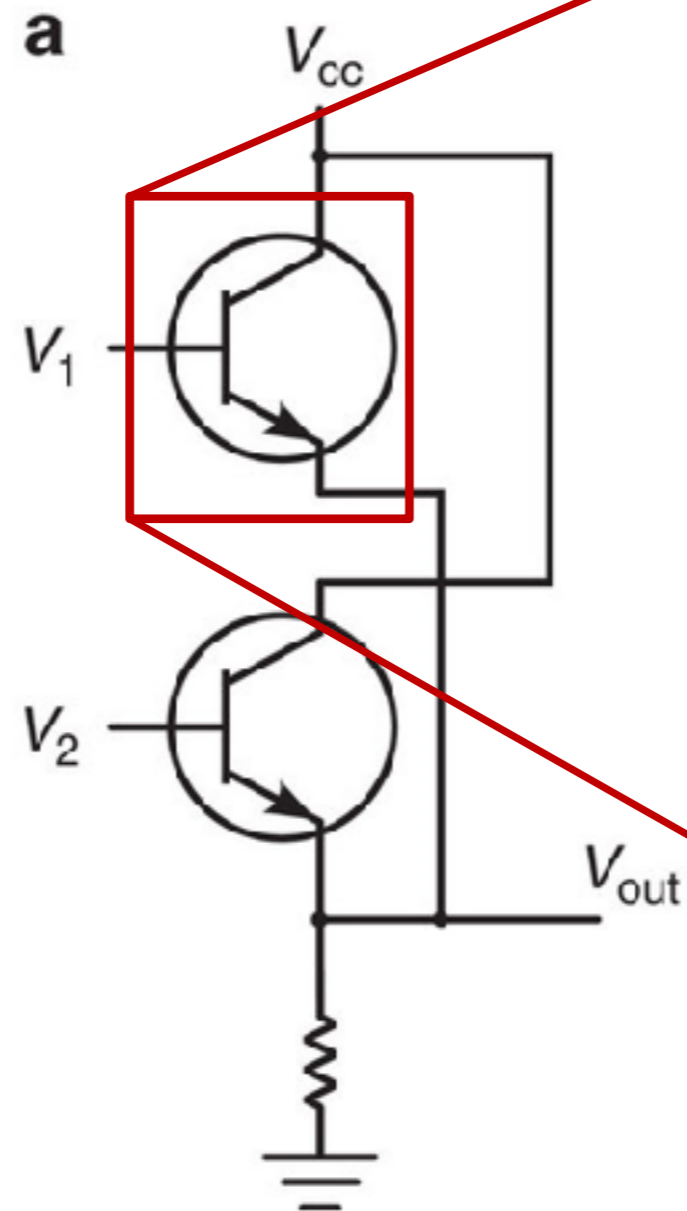
INGRESSO		USCITA
A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1



# Back to the real world....

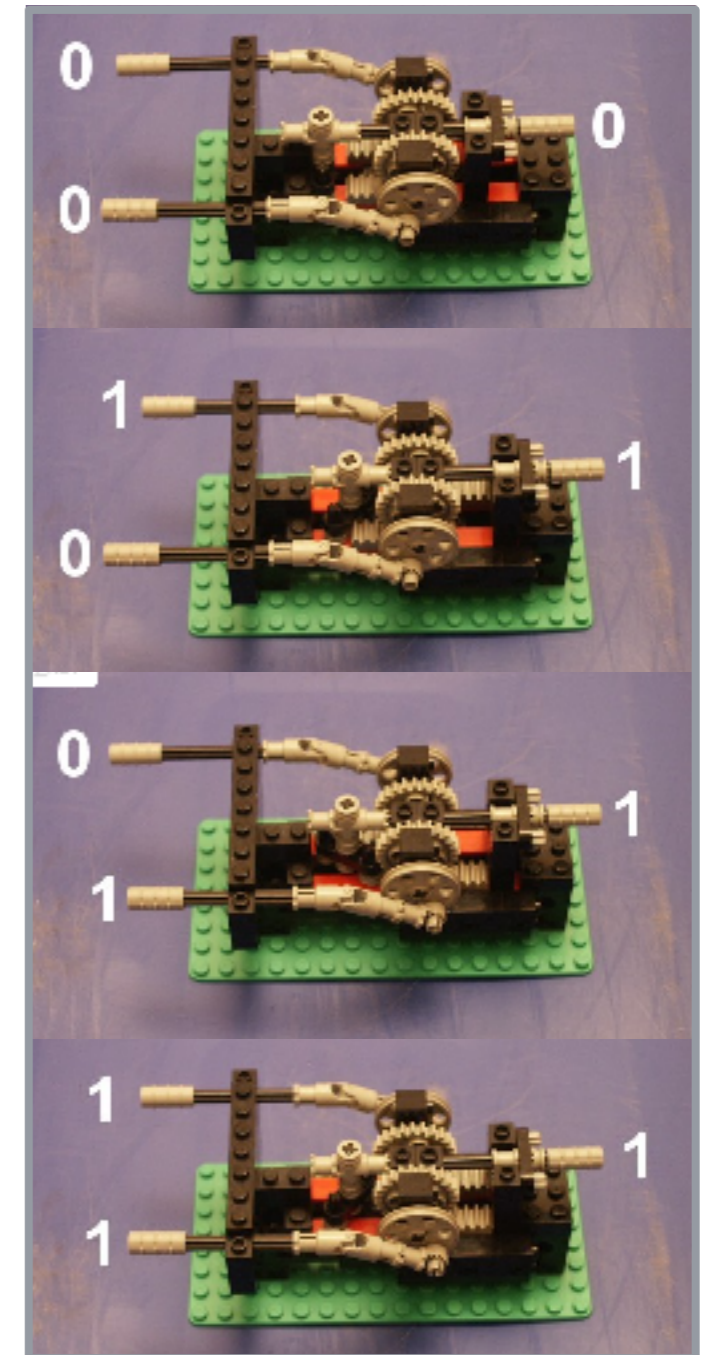
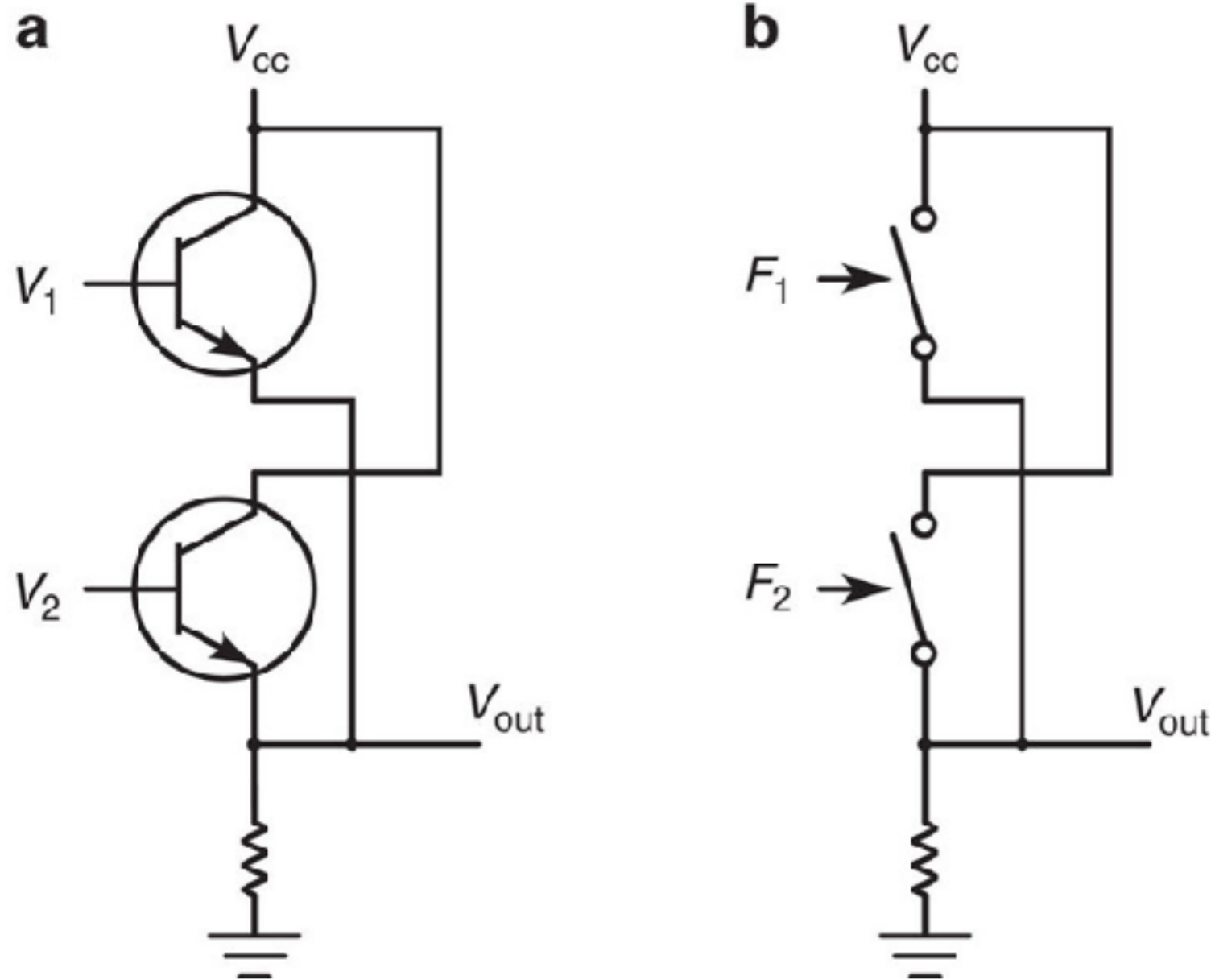
## OR gate

INGRESSO		USCITA
A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

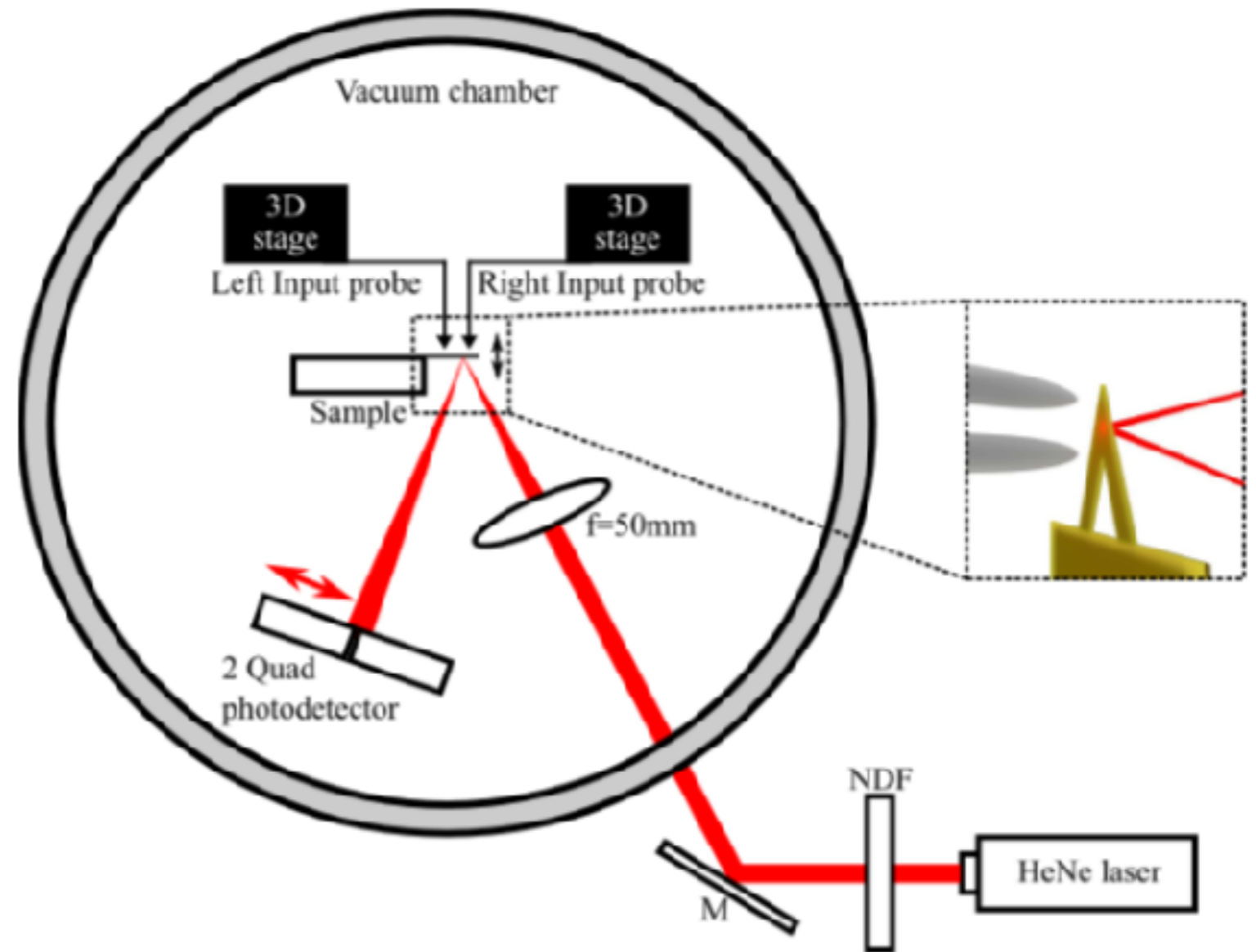
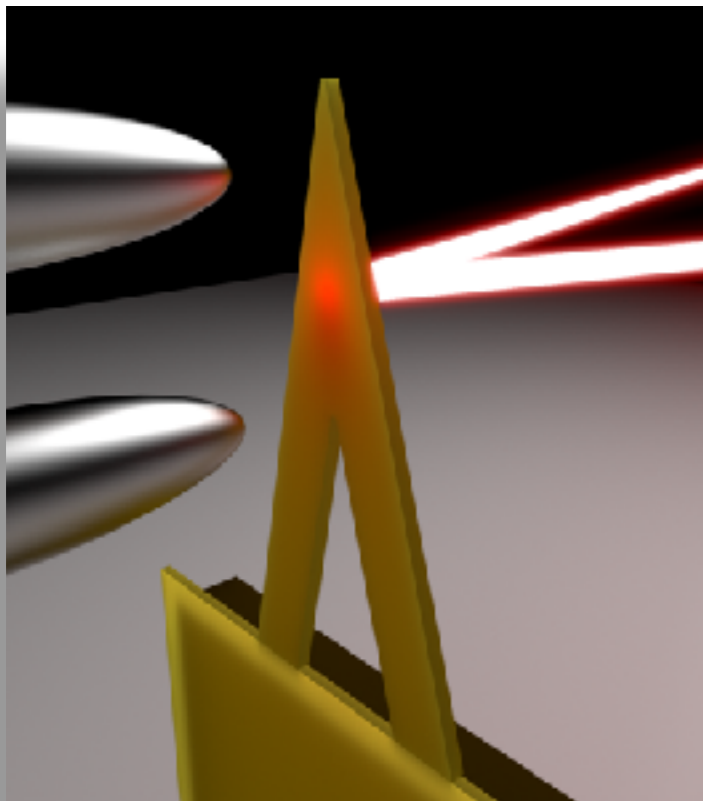


# Back to the real world....

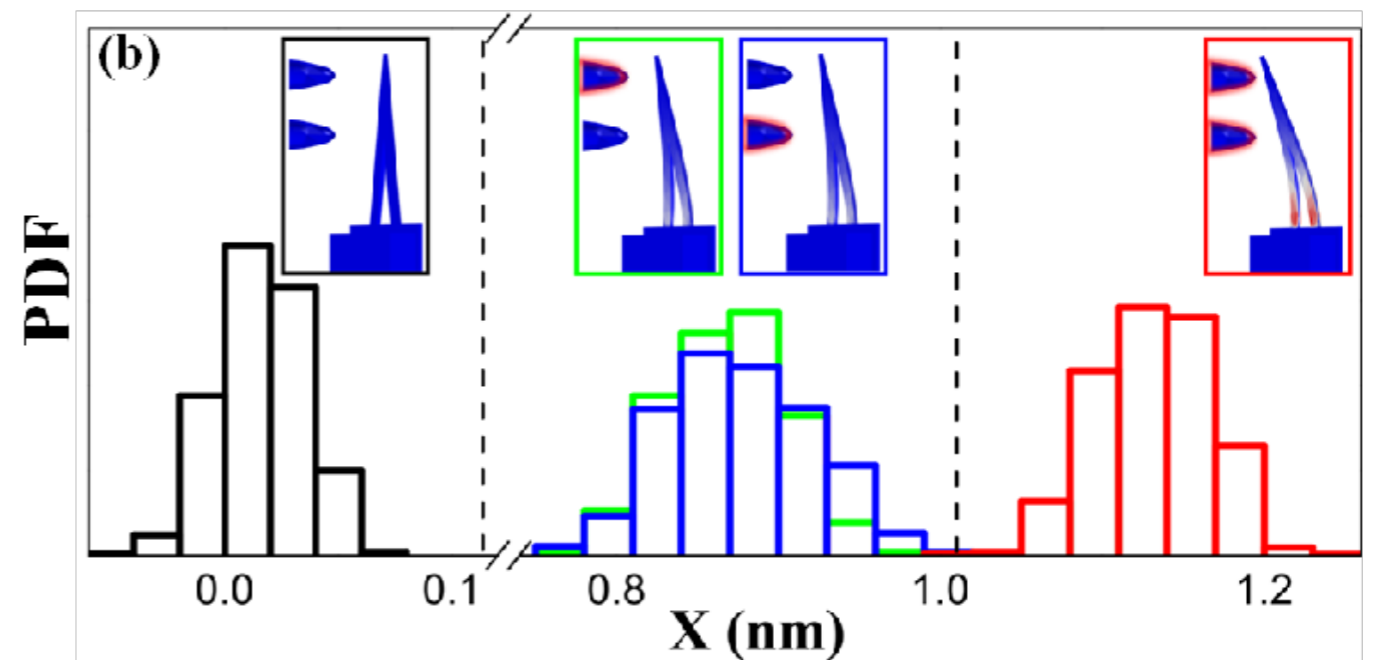
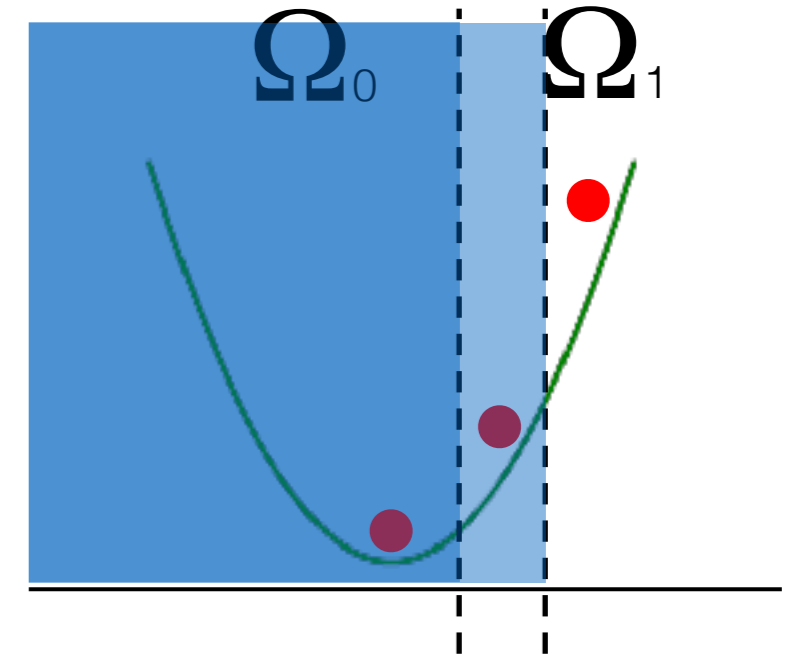
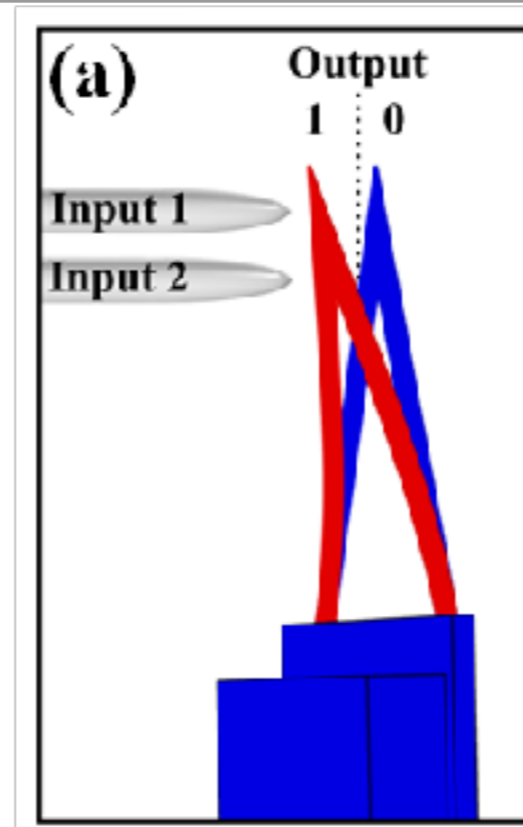
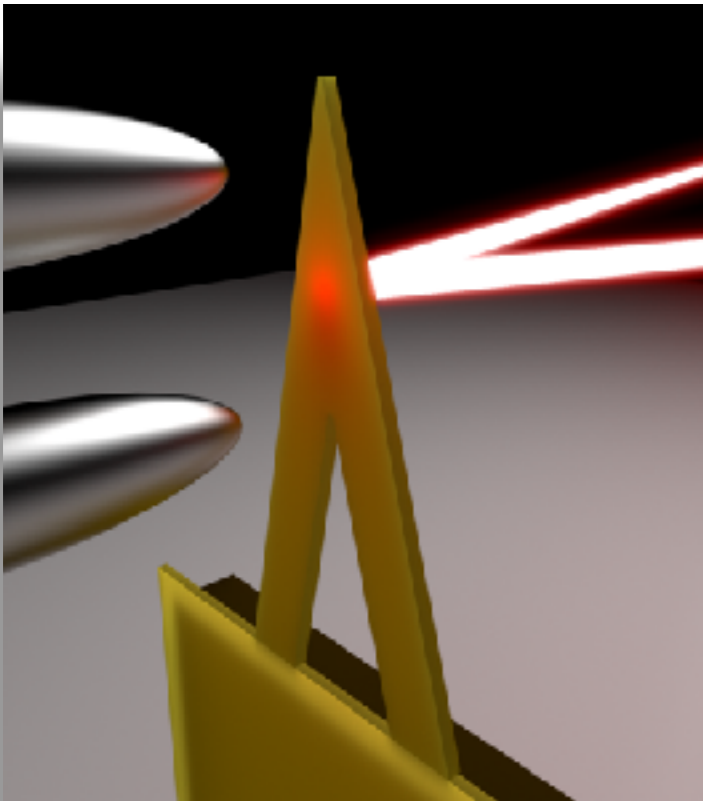
## OR gate



# The experimental setup

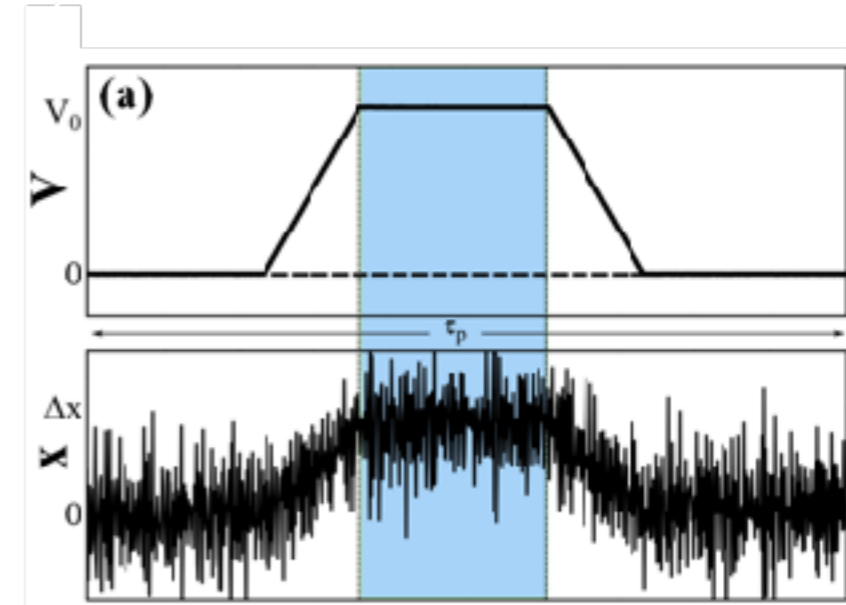
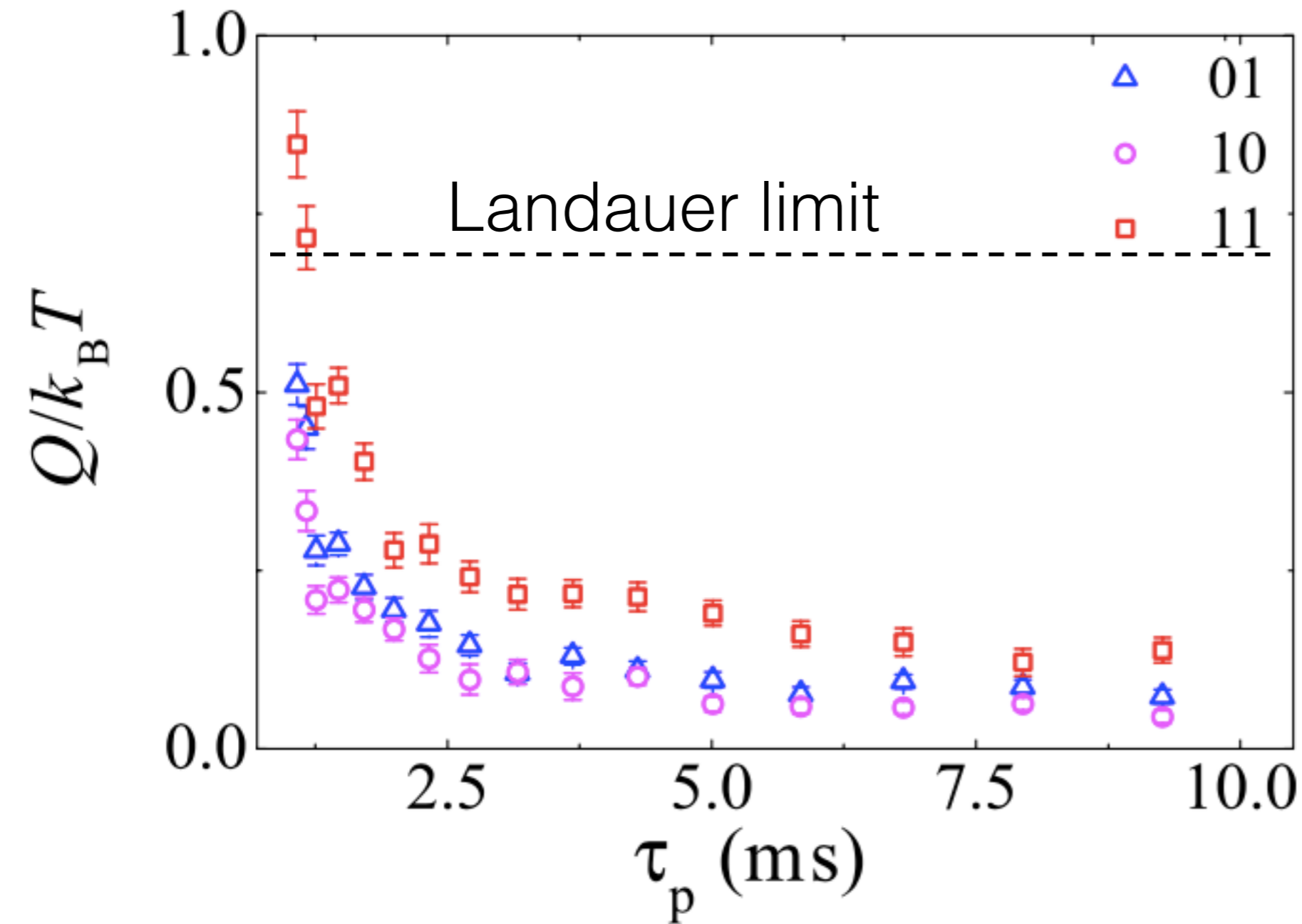


# The experimental setup

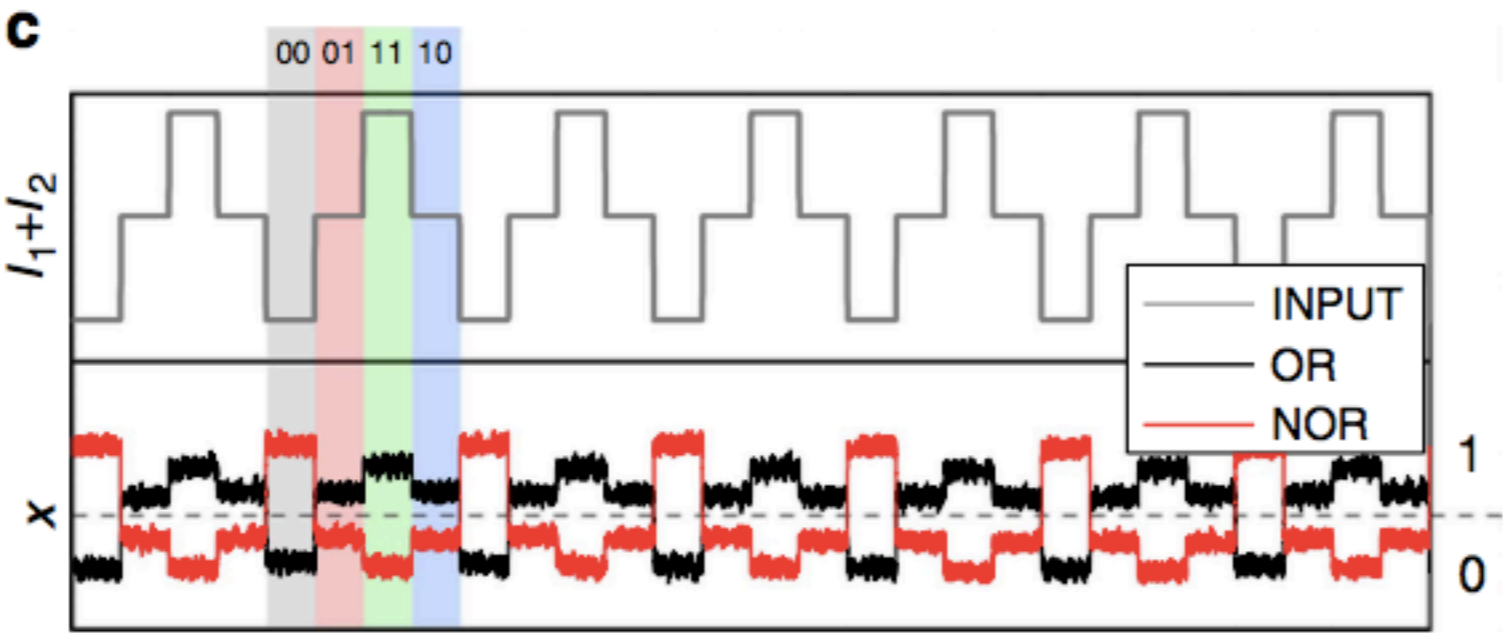
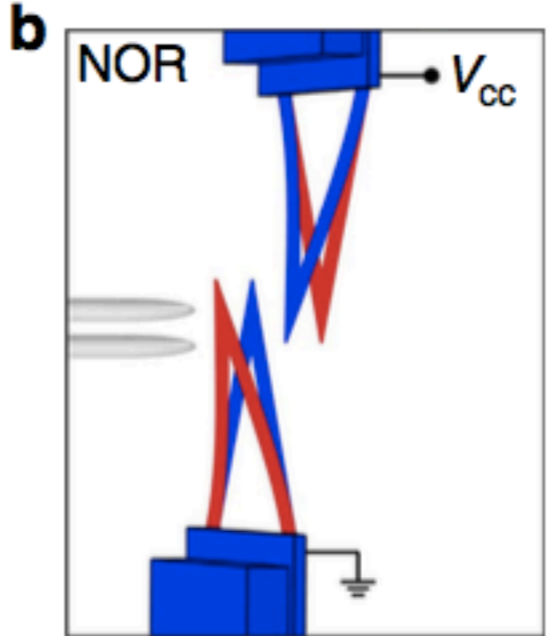
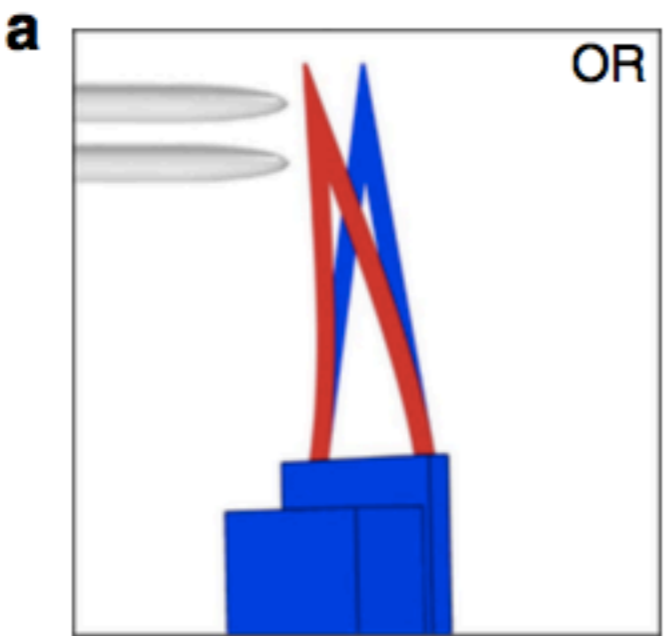




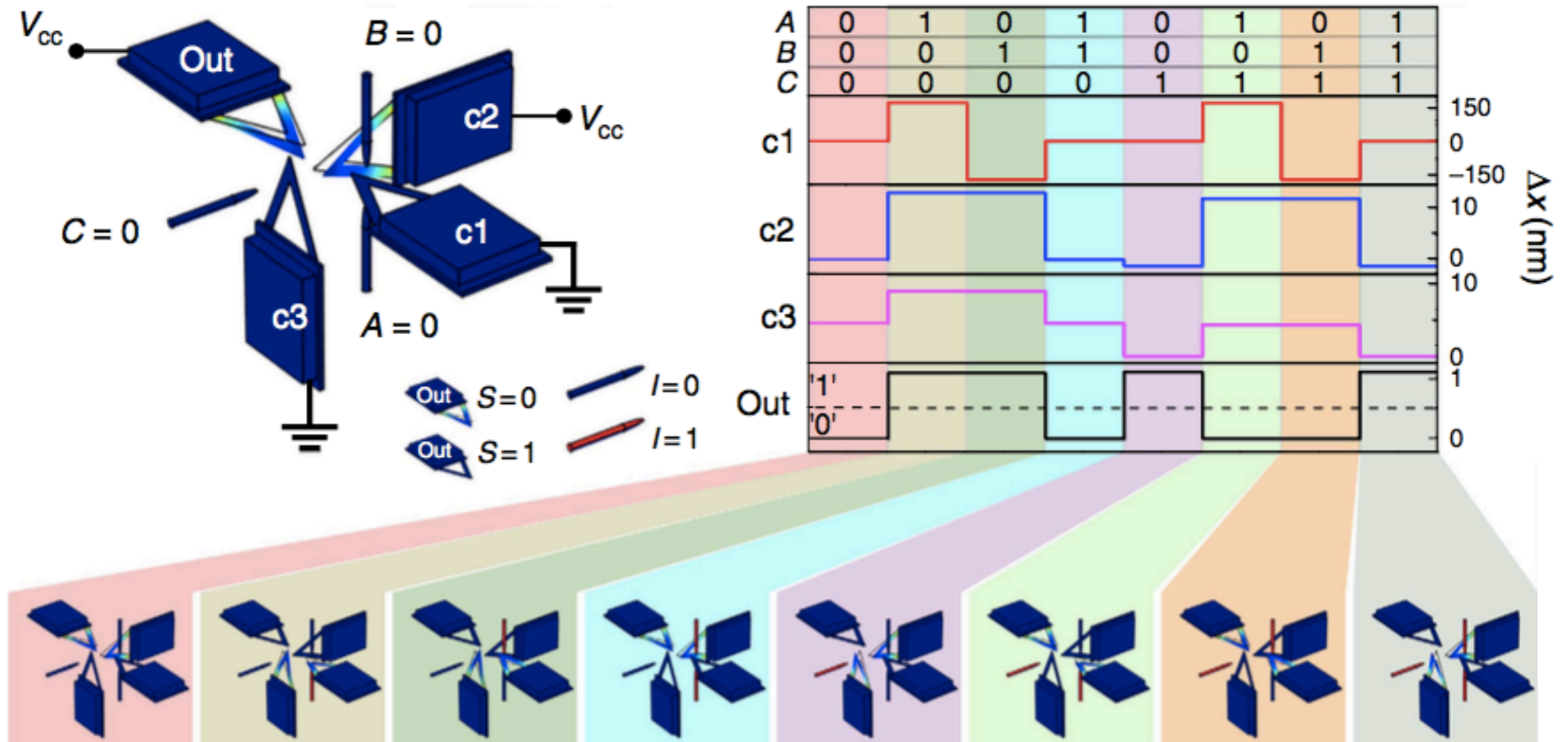
# The experimental setup



# XOR gate

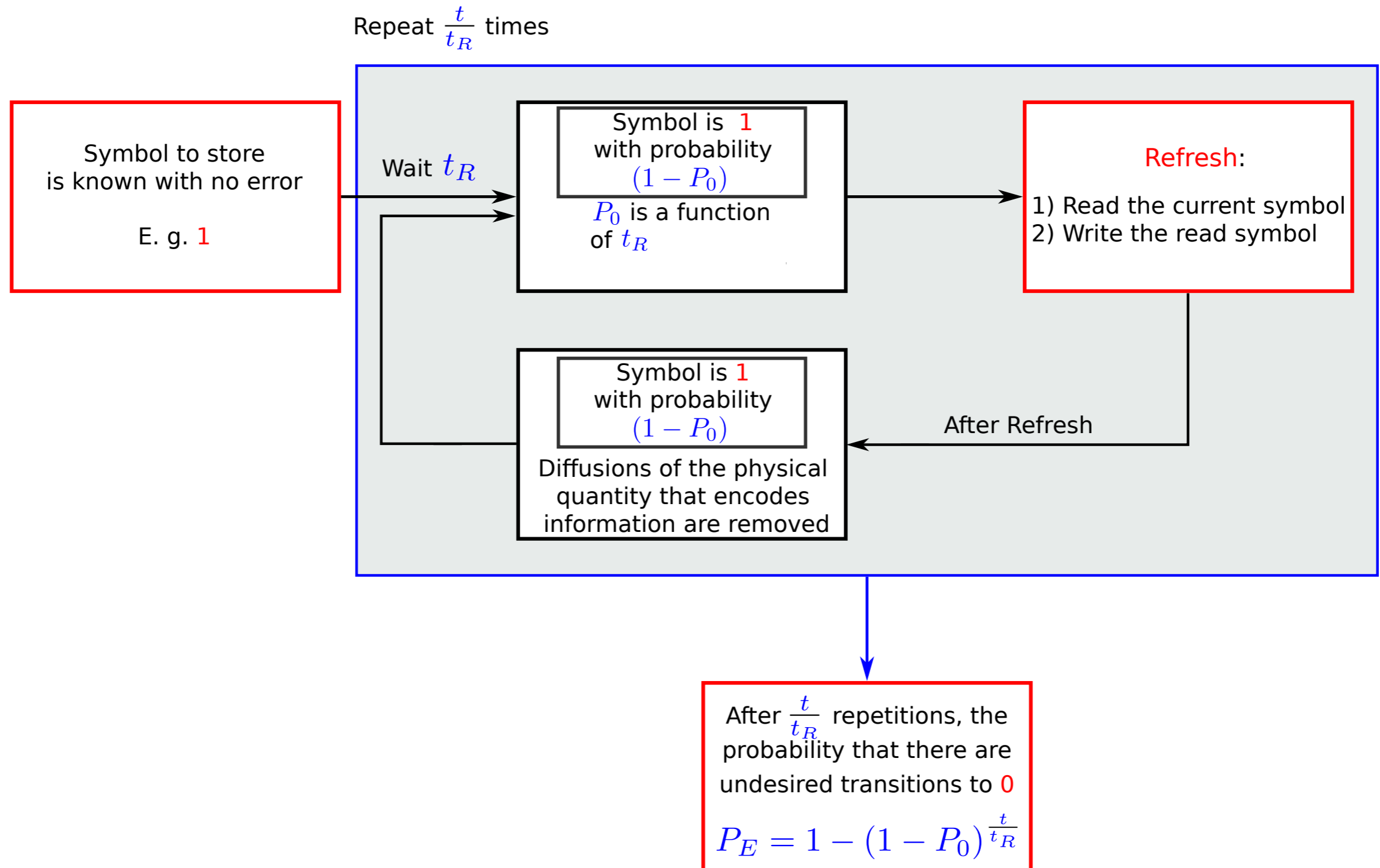


# Full adder



Minimum energy consumption for memory  
preservation

# The refresh procedure



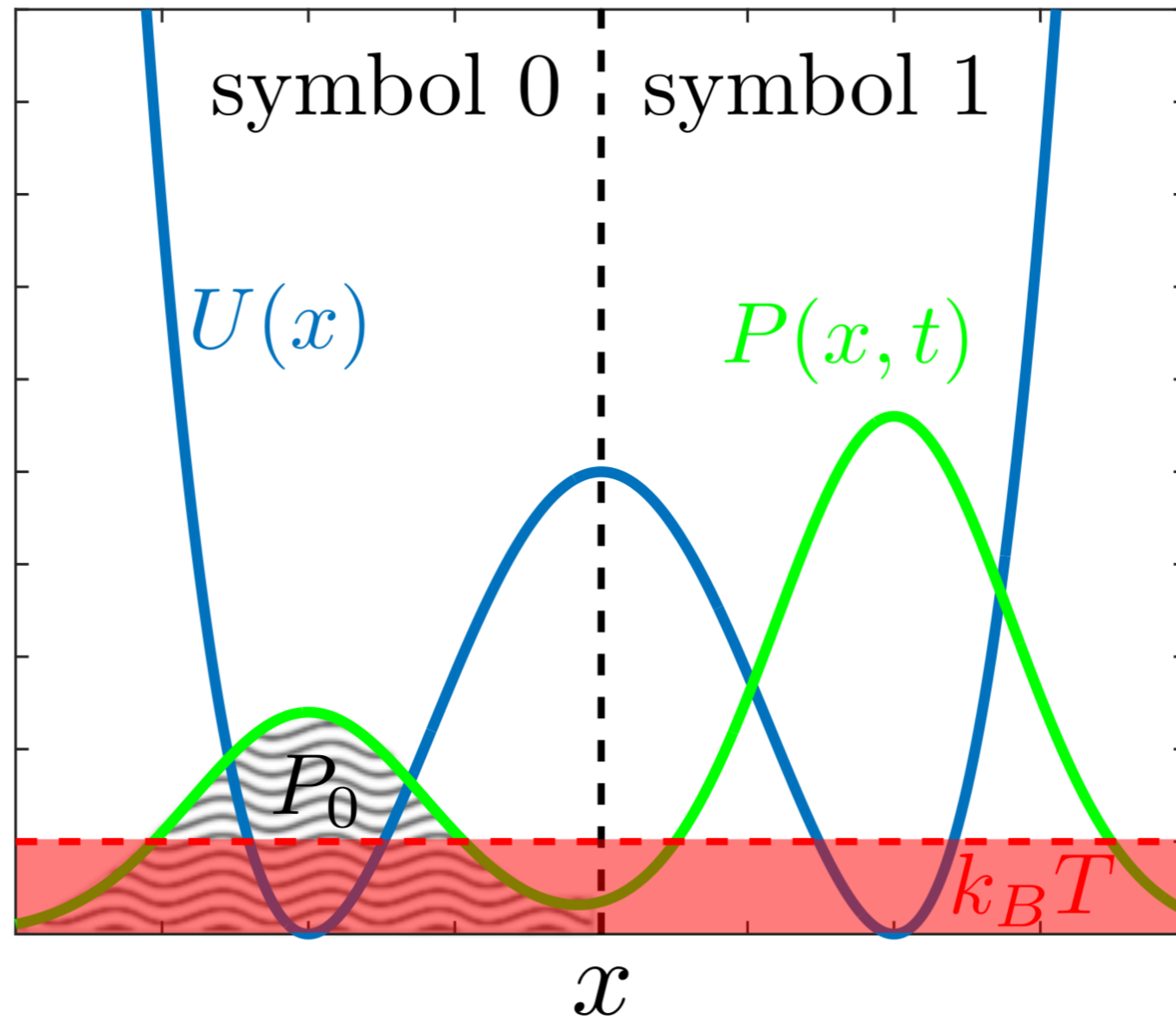
To evaluate the energy cost of the refresh procedure we need:

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- A physical description of the memory
- A characterisation of  $P_0$  as function of refresh time  $t_R$
- A physical description of the refresh procedure
- A characterisation of total error probability  $P_E$  as function of refresh time  $t_R$  after a fixed time

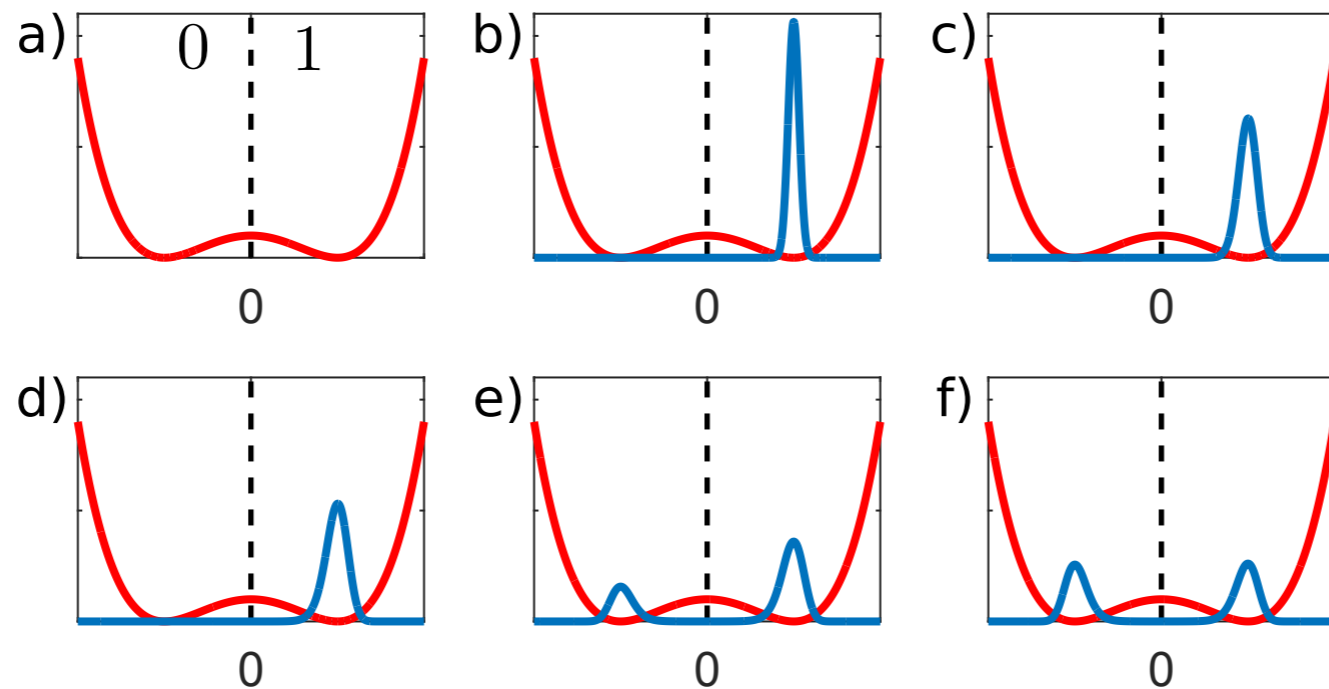
# Physical description of the memory

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# Characterisation of $P_0$ as function of refresh time

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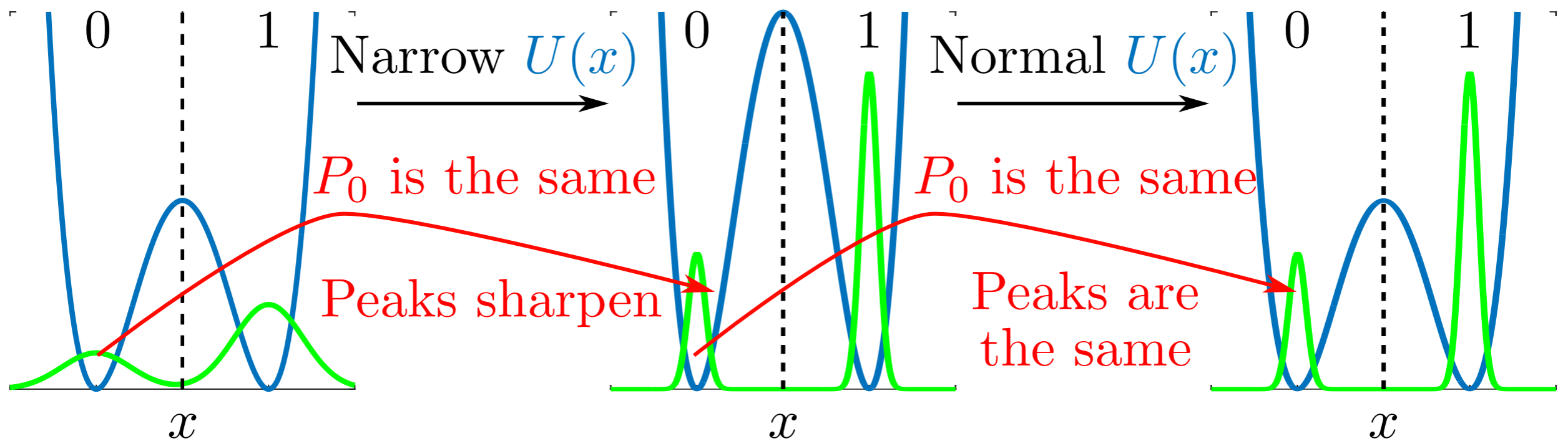
$$\frac{\partial}{\partial t} p(x, t) = \frac{\partial}{\partial x} \left( \frac{\partial U}{\partial x} p(x, t) \right) + \frac{k_B T}{\Delta U} \frac{\partial^2}{\partial x^2} p(x, t),$$

$$P_0(t) = \int_{-\infty}^{\infty} p(x, t) dx$$



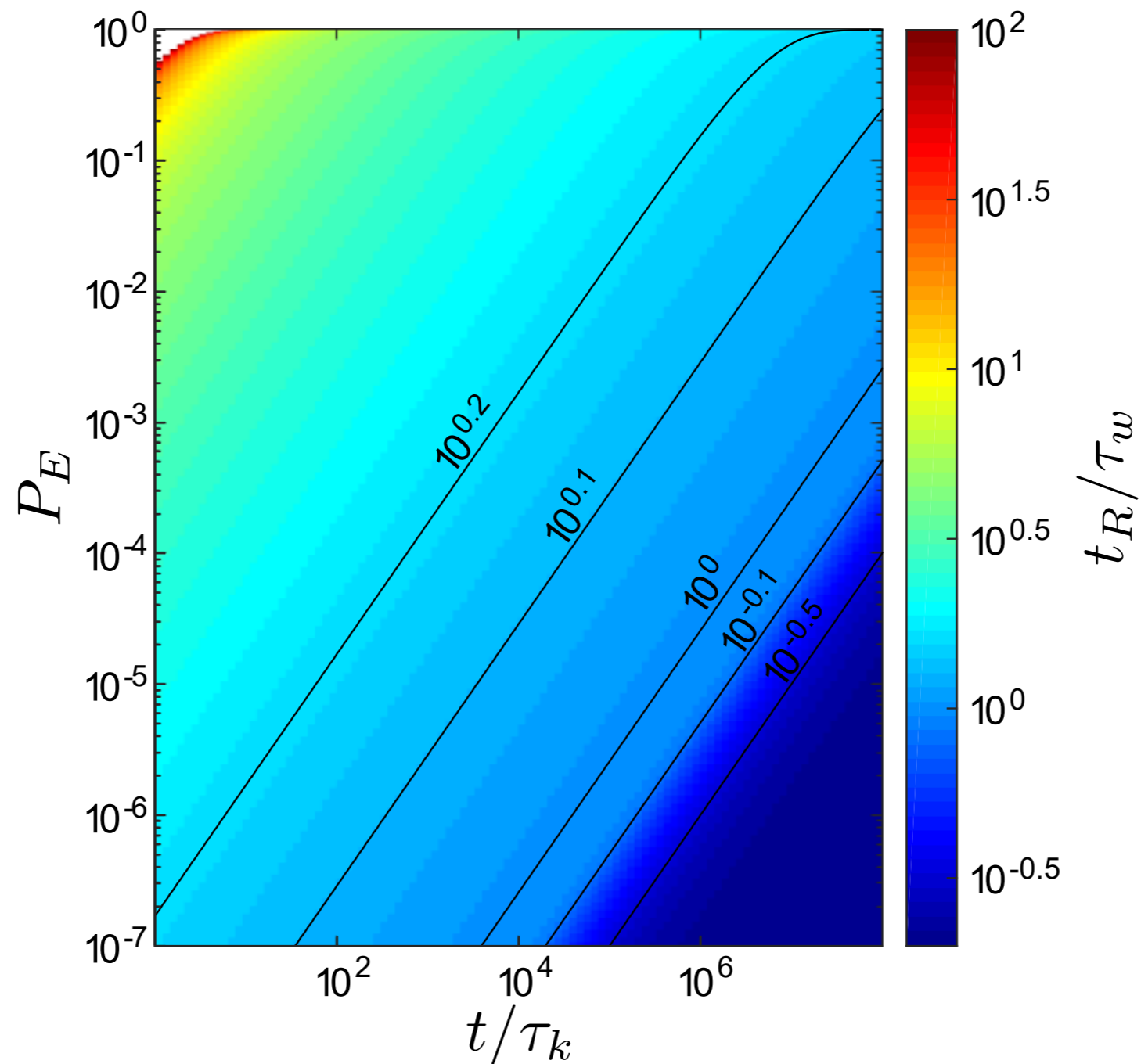
# Physical description of the refresh procedure

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# Characterisation of $P_E$ as function of refresh time

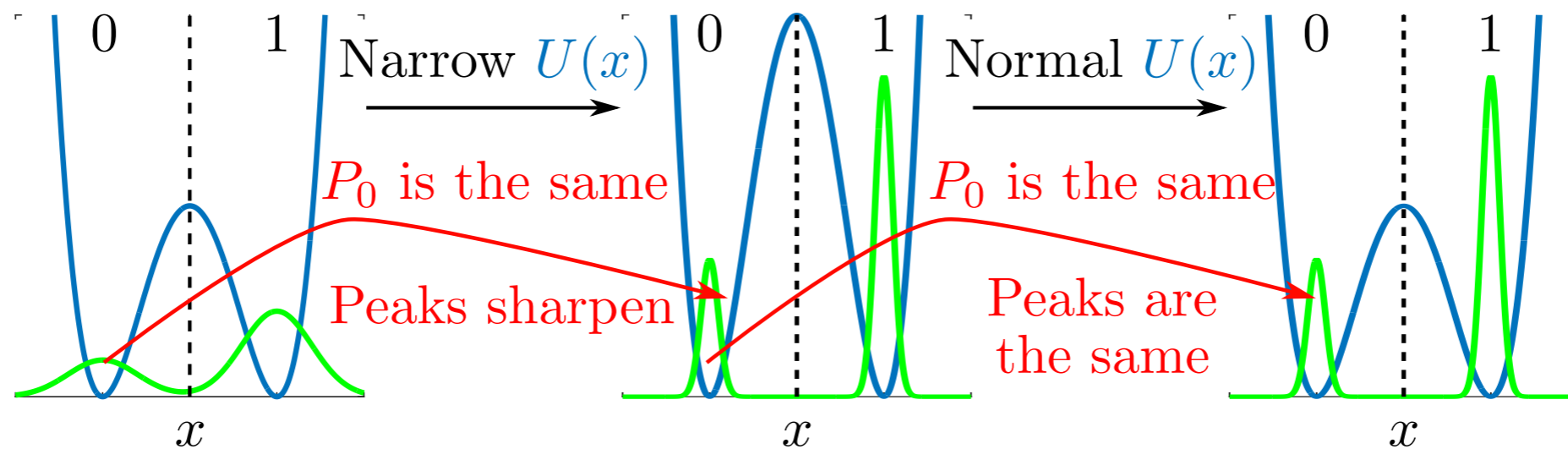
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What is the fundamental cost for preserving a memory for a fixed time with a given probability of error?

# Study of the energy cost of refresh procedure

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- ▶ Considering the harmonic approximation inside each well the refresh operation changes:

$$\sigma(t_R) = \sqrt{\sigma_w^2 + \exp\left(-\frac{t_R}{\tau_w}\right) (\sigma_i^2 - \sigma_w^2)} \quad \text{in } \sigma_i$$

Minimum energy required to preserve a memory over a fixed time with a given error probability

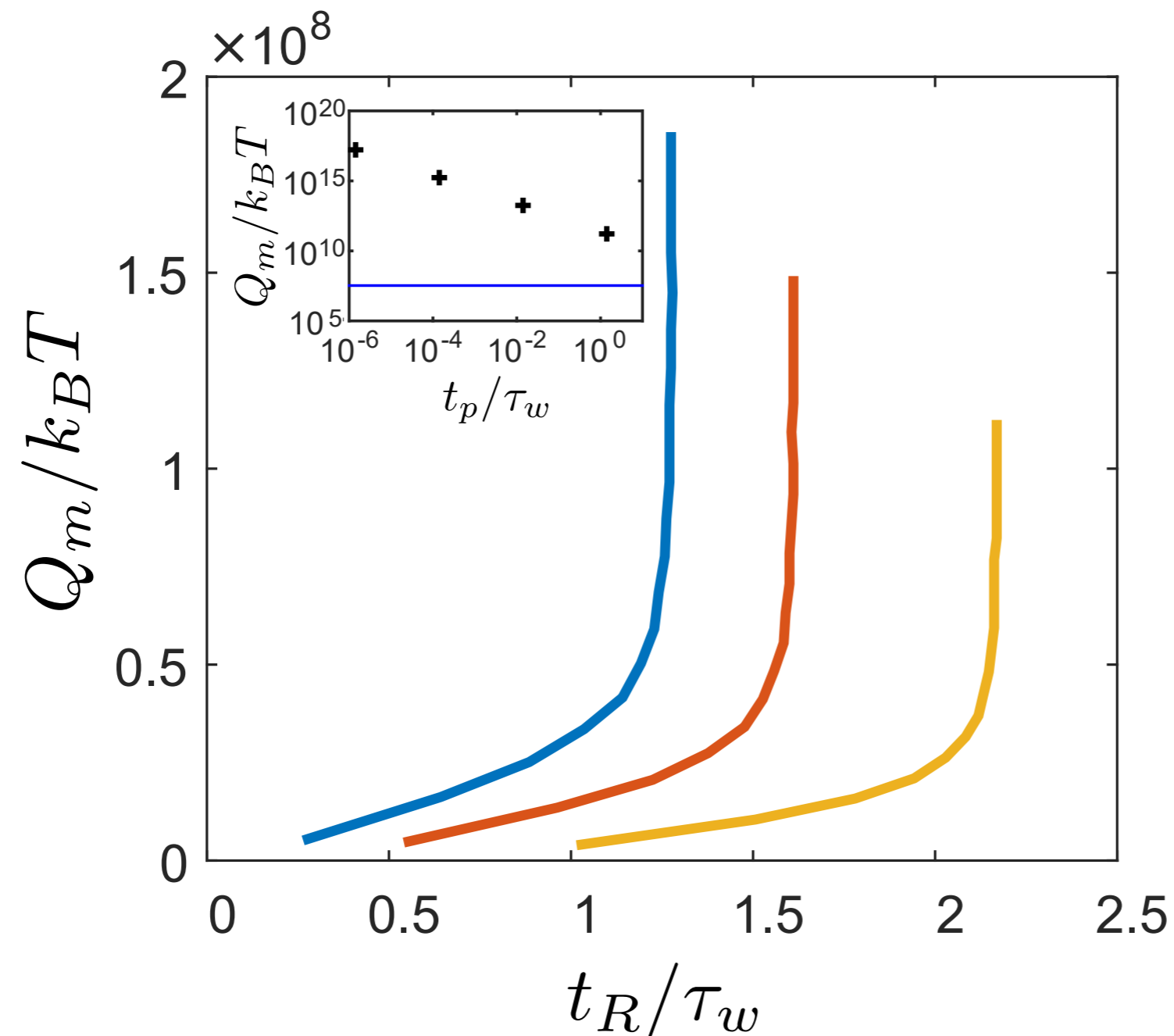
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$$Q_m = -NT\Delta S = \frac{\bar{t}}{t_R} k_B T \ln \left( \frac{\sqrt{(\sigma_w^2 + e^{-\frac{t_R}{\tau_w}} (\sigma_i^2 - \sigma_w^2))}}{\sigma_i} \right)$$

Minimum energy required to preserve a memory over a fixed time with a given error probability

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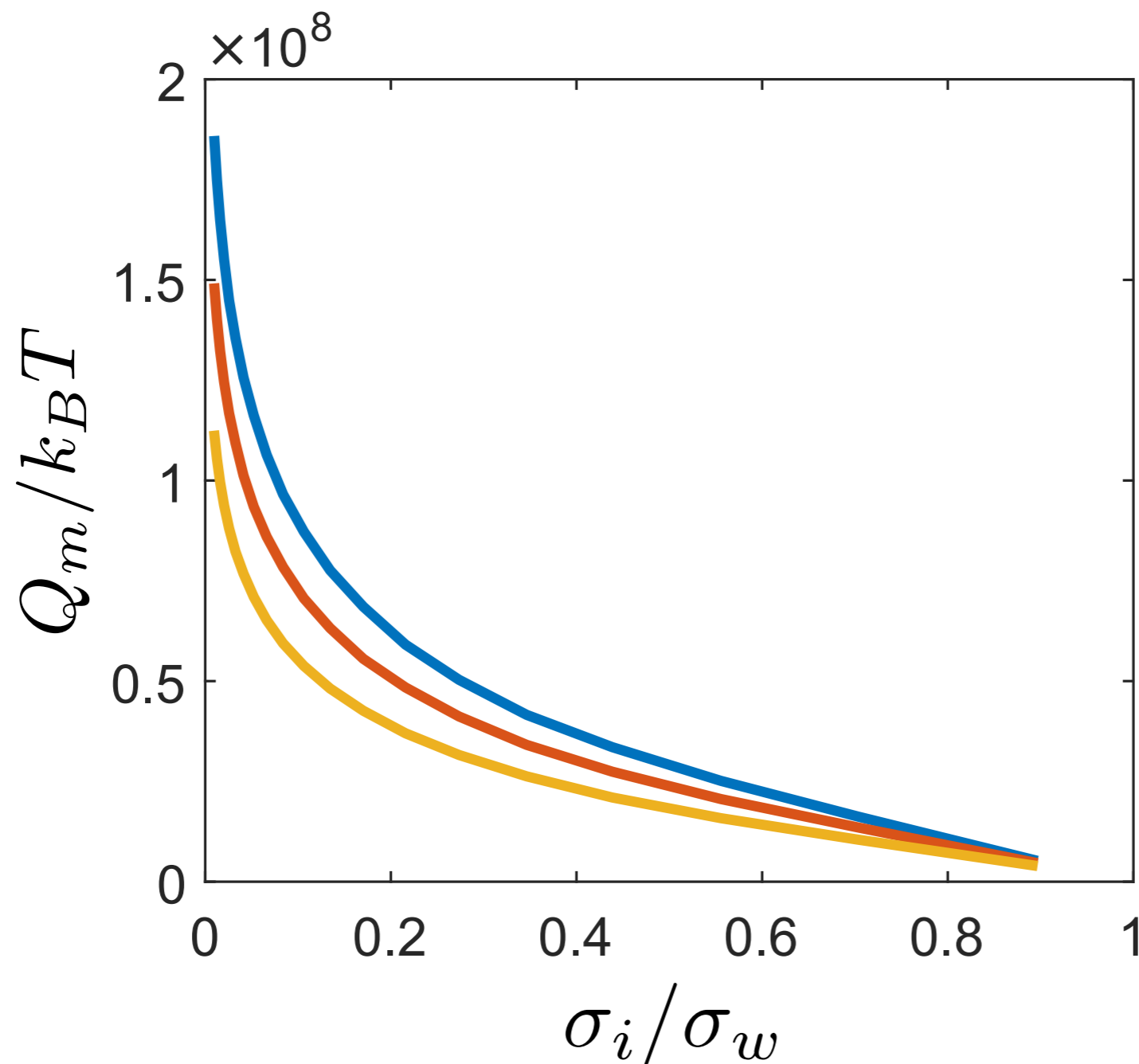
$P_E=1 \times 10^{-6}$   $P_E=1 \times 10^{-4}$   $P_E=1 \times 10^{-2}$



Minimum energy required to preserve a memory over a fixed time with a given error probability

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$P_E=1 \times 10^{-6}$   $P_E=1 \times 10^{-4}$   $P_E=1 \times 10^{-2}$



# Limits to computation

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- Minimum size of computing device
- Maximum computational speed of a self-contained system
- Information storage in a finite volume
- Energy consumption limit to:
  - computation
  - memory preservation



# References

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# Thank you for your attention!

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Noise in Physical Systems

